

nature, or (ii) not financial in nature but is authorized to open an account at the Federal Reserve Bank of New York. We consider each of the following entities to be a single FIMA entity:

(1) A foreign central bank or regional central bank.

(2) A foreign governmental monetary or finance entity.

(3) A non-governmental international financial organization that is not private in nature (for example, the International Monetary Fund, the World Bank, the Inter-American Development Bank, and the Asian Development Bank).

(4) A non-financial international organization that the United States participates in (for example, the United Nations).

(5) A multi-party arrangement of a governmental ministry and/or a foreign central bank or monetary authority with a United States Government Department and/or the Federal Reserve Bank of New York.

(6) A foreign or international monetary entity or an entity authorized by statute or by us to open accounts at the Federal Reserve Bank of New York.

(g) *Other Bidder*—We do not consider a bidder defined by any of the above categories to be a bidder in this category. For purposes of this definition, “other bidder” means an institution or organization with a unique IRS-assigned employer identification number. This definition includes such entities as an association, church, university, union, or club. This category does not include any person or entity acting in a fiduciary or investment management capacity, a sole proprietorship, an investment account, an investment fund, a form of registration, or investment ownership designation.

## II. HOW TO OBTAIN SEPARATE BIDDER RECOGNITION

Under certain circumstances, we may recognize a major organizational component (e.g., the parent or a subsidiary) in a corporate or partnership structure as a bidder separate from the larger corporate or partnership structure. We also may recognize two or more major organizational components collectively as one bidder. All of the following criteria must be met for such component(s) to qualify for recognition as a separate bidder:

(a) Such component(s) must be prohibited by law or regulation from exchanging, or must have established written internal procedures designed to prevent the exchange of, information related to bidding in Treasury securities with any other component in the corporate or partnership structure;

(b) Such component(s) must not be created for the purpose of circumventing our bidding and award limitations;

(c) Decisions related to purchasing Treasury securities at auction and participation

in specific auctions must be made by employees of such component(s). Employees of such component(s) that make decisions to purchase or dispose of Treasury securities must not perform the same function for other components within the corporate or partnership structure; and

(d) The records of such component(s) related to the bidding for, acquisition of, and disposition of Treasury securities must be maintained by such component(s). Those records must be identifiable—separate and apart from similar records for other components within the corporate or partnership structure. To obtain recognition as a separate bidder, each component or group of components must request such recognition from us, provide a description of the component or group and its position within the corporate or partnership structure, and provide the following certification:

[Name of the bidder] hereby certifies that to the best of its knowledge and belief it meets the criteria for a separate bidder as described in appendix A to 31 CFR part 356. The above-named bidder also certifies that it has established written policies or procedures, including ongoing compliance monitoring processes, that are designed to prevent the component or group of components from:

(1) Exchanging any of the following information with any other part of the corporate [partnership] structure: (a) Yields, discount rates, or discount margins at which it plans to bid; (b) amounts of securities for which it plans to bid; (c) positions that it holds or plans to acquire in a security being auctioned; and (d) investment strategies that it plans to follow regarding the security being auctioned, or

(2) In any way intentionally acting together with any other part of the corporate [partnership] structure with respect to formulating or entering bids in a Treasury auction.

The above-named bidder agrees that it will promptly notify the Department in writing when any of the information provided to obtain separate bidder status changes or when this certification is no longer valid.

[69 FR 45202, July 28, 2004, as amended at 70 FR 29456, May 23, 2005; 78 FR 46430, July 31, 2013]

## APPENDIX B TO PART 356—FORMULAS AND TABLES

I. Computation of Interest on Treasury Bonds and Notes.

II. Formulas for Conversion of Non-indexed Security Yields to Equivalent Prices.

III. Formulas for Conversion of Inflation-Protected Security Yields to Equivalent Prices.

## IV. Formulas for Conversion of Floating Rate Note Discount Margins to Equivalent Prices

## V. Computation of Adjusted Values and Payment Amounts for Stripped Inflation-Protected Interest Components.

## VI. Computation of Purchase Price, Discount Rate, and Investment Rate (Coupon-Equivalent Yield) for Treasury Bills.

The examples in this appendix are given for illustrative purposes only and are in no way a prediction of interest rates on any bills, notes, or bonds issued under this part. In some of the following examples, we use intermediate rounding for ease in following the calculations. In actual practice, we generally do not round prior to determining the final result.

If you use a multi-decimal calculator, we recommend setting your calculator to at least 13 decimals and then applying normal rounding procedures. This should be sufficient to obtain the same final results. However, in the case of any discrepancies, our determinations will be final.

## I. COMPUTATION OF INTEREST ON TREASURY BONDS AND NOTES

## A. Treasury Non-indexed Securities

1. *Regular Half-Year Payment Period.* We pay interest on marketable Treasury non-indexed securities on a semiannual basis. The regular interest payment period is a full half-year of six calendar months. Examples of half-year periods are: (1) February 15 to August 15, (2) May 31 to November 30, and (3) February 29 to August 31 (in a leap year). Calculation of an interest payment for a non-indexed note with a par amount of \$1,000 and an interest rate of 8% is made in this manner:  $(\$1,000 \times .08)/2 = \$40$ . Specifically, a semiannual interest payment represents one half of one year's interest, and is computed on this basis regardless of the actual number of days in the half-year.

2. *Daily Interest Decimal.* We compute a daily interest decimal in cases where an interest payment period for a non-indexed security is shorter or longer than six months or where accrued interest is payable by an investor. We base the daily interest decimal on the actual number of calendar days in the half-year or half-years involved. The number of days in any half-year period is shown in Table 1.

TABLE 1

Interest period	Beginning and ending days are 1st or 15th of the months listed under interest period (number of days)		Beginning and ending days are the last days of the months listed under interest period (number of days)	
	Regular year	Leap year	Regular year	Leap year
January to July .....	181	182	181	182
February to August .....	181	182	184	184
March to September .....	184	184	183	183
April to October .....	183	183	184	184
May to November .....	184	184	183	183
June to December .....	183	183	184	184
July to January .....	184	184	184	184
August to February .....	184	184	181	182
September to March .....	181	182	182	183
October to April .....	182	183	181	182
November to May .....	181	182	182	183
December to June .....	182	183	181	182

Table 2 below shows the daily interest decimals covering interest from  $\frac{1}{8}\%$  to 20% on \$1,000 for one day in increments of  $\frac{1}{8}\%$  of one percent. These decimals represent  $\frac{1}{181}$ ,

$\frac{1}{182}$ ,  $\frac{1}{183}$ , or  $\frac{1}{184}$  of a full semiannual interest payment, depending on which half-year is applicable.

TABLE 2

[Decimal for one day's interest on \$1,000 at various rates of interest, payable semiannually or on a semiannual basis, in regular years of 365 days and in years of 366 days (to determine applicable number of days, see table 1.)]

Rate per annum (percent)	Half-year of 184 days	Half-year of 183 days	Half-year of 182 days	Half-year of 181 days
$\frac{1}{8}$ .....	0.003396739	0.003415301	0.003434066	0.003453039
$\frac{1}{4}$ .....	0.006793478	0.006830601	0.006868132	0.006906077
$\frac{3}{8}$ .....	0.010190217	0.010245902	0.010302198	0.010359116
$\frac{1}{2}$ .....	0.013586957	0.013661202	0.013736264	0.013812155

TABLE 2—Continued

[Decimal for one day's interest on \$1,000 at various rates of interest, payable semiannually or on a semiannual basis, in regular years of 365 days and in years of 366 days (to determine applicable number of days, see table 1.)]

Rate per annum (percent)	Half-year of 184 days	Half-year of 183 days	Half-year of 182 days	Half-year of 181 days
5/8 .....	0.016983696	0.017076503	0.017170330	0.017265193
3/4 .....	0.020380435	0.020491803	0.020604396	0.020718232
7/8 .....	0.023777174	0.023907104	0.024038462	0.024171271
1 .....	0.027173913	0.027322404	0.027472527	0.027624309
1 1/8 .....	0.030570652	0.030737705	0.030906593	0.031077348
1 1/4 .....	0.033967391	0.034153005	0.034340659	0.034530387
1 3/8 .....	0.037364130	0.037568306	0.037774725	0.037983425
1 1/2 .....	0.040760870	0.040983607	0.041208791	0.041436464
1 5/8 .....	0.044157609	0.044398907	0.044642857	0.044889503
1 3/4 .....	0.047554348	0.047814208	0.048076923	0.048342541
1 7/8 .....	0.050951087	0.051229508	0.051510989	0.051795580
2 .....	0.054347826	0.054644809	0.054945055	0.055248619
2 1/8 .....	0.057744565	0.058060109	0.058379121	0.058701657
2 1/4 .....	0.061141304	0.061475410	0.061813187	0.062154696
2 3/8 .....	0.064538043	0.064890710	0.065247253	0.065607735
2 1/2 .....	0.067934783	0.068306011	0.068681319	0.069060773
2 5/8 .....	0.071331522	0.071721311	0.072115385	0.072513812
2 3/4 .....	0.074728261	0.075136612	0.075549451	0.075966851
2 7/8 .....	0.078125000	0.078551913	0.078983516	0.079419890
3 .....	0.081521739	0.081967213	0.082417582	0.082872928
3 1/8 .....	0.084918478	0.085382514	0.085851648	0.086325967
3 1/4 .....	0.088315217	0.088797814	0.089285714	0.089779006
3 3/8 .....	0.091711957	0.092213115	0.092719780	0.093232044
3 1/2 .....	0.095108696	0.095628415	0.096153846	0.096685083
3 5/8 .....	0.098505435	0.099043716	0.099587912	0.100138122
3 3/4 .....	0.101902174	0.102459016	0.103021978	0.103591160
3 7/8 .....	0.105298913	0.105874317	0.106456044	0.107044199
4 .....	0.108695652	0.109289617	0.109890110	0.110497238
4 1/8 .....	0.112092391	0.112704918	0.113324176	0.113950276
4 1/4 .....	0.115489130	0.116120219	0.116758242	0.117403315
4 3/8 .....	0.118885870	0.119535519	0.120192308	0.120856354
4 1/2 .....	0.122282609	0.122950820	0.123626374	0.124309392
4 5/8 .....	0.125679348	0.126366120	0.127060440	0.127762431
4 3/4 .....	0.129076087	0.129781421	0.130494505	0.131215470
4 7/8 .....	0.132472826	0.133196721	0.133928571	0.134668508
5 .....	0.135869565	0.136612022	0.137362637	0.138121547
5 1/8 .....	0.139266304	0.140027322	0.140796703	0.141574586
5 1/4 .....	0.142663043	0.143442623	0.144230769	0.145027624
5 3/8 .....	0.146059783	0.146857923	0.147664835	0.148480663
5 1/2 .....	0.149456522	0.150273224	0.151098901	0.151933702
5 5/8 .....	0.152853261	0.153688525	0.154532967	0.155386740
5 3/4 .....	0.156250000	0.157103825	0.157967033	0.158839779
5 7/8 .....	0.159646739	0.160519126	0.161401099	0.162292818
6 .....	0.163043478	0.163934426	0.164835165	0.165745856
6 1/8 .....	0.166440217	0.167349727	0.168269231	0.169198895
6 1/4 .....	0.169836957	0.170765027	0.171703297	0.172651934
6 3/8 .....	0.173233696	0.174180328	0.175137363	0.176104972
6 1/2 .....	0.176630435	0.177595628	0.178571429	0.179558011
6 5/8 .....	0.180027174	0.181010929	0.182005495	0.183011050
6 3/4 .....	0.183423913	0.184426230	0.185439560	0.186464088
6 7/8 .....	0.186820652	0.187841530	0.188873626	0.189917127
7 .....	0.190217391	0.191256831	0.192307692	0.193370166
7 1/8 .....	0.193614130	0.194672131	0.195741758	0.196823204
7 1/4 .....	0.197010870	0.198087432	0.199175824	0.200276243
7 3/8 .....	0.200407609	0.201502732	0.202609890	0.203729282
7 1/2 .....	0.203804348	0.204918033	0.206043956	0.207182320
7 5/8 .....	0.207201087	0.208333333	0.209478022	0.210635359
7 3/4 .....	0.210597826	0.211748634	0.212912088	0.214088398
7 7/8 .....	0.213994565	0.215163934	0.216346154	0.217541436
8 .....	0.217391304	0.218579235	0.219780220	0.220994475
8 1/8 .....	0.220788043	0.221994536	0.223214286	0.224447514
8 1/4 .....	0.224184783	0.225409836	0.226646352	0.227900552
8 3/8 .....	0.227581522	0.228825137	0.230082418	0.231353591
8 1/2 .....	0.230978261	0.232240437	0.233516484	0.234806630
8 5/8 .....	0.234375000	0.235655738	0.236950549	0.238259669
8 3/4 .....	0.237771739	0.239071038	0.240384615	0.241712707
8 7/8 .....	0.241168478	0.242486339	0.243818681	0.245165746
9 .....	0.244565217	0.245901639	0.247252747	0.248618785
9 1/8 .....	0.247961957	0.249316940	0.250686813	0.252071823

TABLE 2—Continued

[Decimal for one day's interest on \$1,000 at various rates of interest, payable semiannually or on a semiannual basis, in regular years of 365 days and in years of 366 days (to determine applicable number of days, see table 1.)]

Rate per annum (percent)	Half-year of 184 days	Half-year of 183 days	Half-year of 182 days	Half-year of 181 days
9¼ .....	0.251358696	0.252732240	0.254120879	0.255524862
9⅜ .....	0.254755435	0.256147541	0.257554945	0.258977901
9½ .....	0.258152174	0.259562842	0.260989011	0.262430939
9⅝ .....	0.261548913	0.262978142	0.264423077	0.265883978
9¾ .....	0.264945652	0.266393443	0.267857143	0.269337017
9⅞ .....	0.268342391	0.269808743	0.271291209	0.272790055
10 .....	0.271739130	0.273224044	0.274725275	0.276243094
10¼ .....	0.275135870	0.276639344	0.278159341	0.279696133
10½ .....	0.278532609	0.280054645	0.281593407	0.283149171
10⅝ .....	0.281929348	0.283469945	0.285027473	0.286602210
10¾ .....	0.285326087	0.286885246	0.288461538	0.290055249
10⅞ .....	0.288722826	0.290300546	0.291895604	0.293508287
10¾ .....	0.292119565	0.293715847	0.295329670	0.296961326
10⅝ .....	0.295516304	0.297131148	0.298763736	0.300414365
11 .....	0.298913043	0.300546448	0.302197802	0.303867403
11¼ .....	0.302309783	0.303961749	0.305631868	0.307320442
11½ .....	0.305706522	0.307377049	0.309065934	0.310773481
11⅝ .....	0.309103261	0.310792350	0.312500000	0.314226519
11¾ .....	0.312500000	0.314207650	0.315934066	0.317679558
11⅝ .....	0.315896739	0.317622951	0.319368132	0.321132597
11¾ .....	0.319293478	0.321038251	0.322802198	0.324585635
11⅝ .....	0.322690217	0.324453552	0.326236264	0.328038674
12 .....	0.326086957	0.327868852	0.329670330	0.331491713
12¼ .....	0.329483696	0.331284153	0.333104396	0.334944751
12½ .....	0.332880435	0.334699454	0.336538462	0.338397790
12⅝ .....	0.336277174	0.338114754	0.339972527	0.341850829
12¾ .....	0.339673913	0.341530055	0.343406593	0.345303867
12⅝ .....	0.343070652	0.344945355	0.346840659	0.348756906
12¾ .....	0.346467391	0.348360656	0.350274725	0.352209945
12⅝ .....	0.349864130	0.351775956	0.353708791	0.355662983
13 .....	0.353260870	0.355191257	0.357142857	0.359116022
13¼ .....	0.356657609	0.358606557	0.360576923	0.362569061
13½ .....	0.360054348	0.362021858	0.364010989	0.366022099
13⅝ .....	0.363451087	0.365437158	0.367445055	0.369475138
13¾ .....	0.366847826	0.368852459	0.370879121	0.372928177
13⅝ .....	0.370244565	0.372267760	0.374313187	0.376381215
13¾ .....	0.373641304	0.375683060	0.377747253	0.379834254
13⅝ .....	0.377038043	0.379098361	0.381181319	0.383287293
14 .....	0.380434783	0.382513661	0.384615385	0.386740331
14¼ .....	0.383831522	0.385928962	0.388049451	0.390193370
14½ .....	0.387228261	0.389344262	0.391483516	0.393646409
14⅝ .....	0.390625000	0.392759563	0.394917582	0.397099448
14¾ .....	0.394021739	0.396174863	0.398351648	0.400552486
14⅝ .....	0.397418478	0.399590164	0.401785714	0.404005525
14¾ .....	0.400815217	0.403005464	0.405219780	0.407458564
14⅝ .....	0.404211957	0.406420765	0.408653846	0.410911602
15 .....	0.407608696	0.409836066	0.412087912	0.414364641
15¼ .....	0.411005435	0.413251366	0.415521978	0.417817680
15½ .....	0.414402174	0.416666667	0.418956044	0.421270718
15⅝ .....	0.417798913	0.420081967	0.422390110	0.424723757
15¾ .....	0.421195652	0.423497268	0.425824176	0.428176796
15⅝ .....	0.424592391	0.426912568	0.429258242	0.431629834
15¾ .....	0.427989130	0.430327869	0.432692308	0.435082873
15⅝ .....	0.431385870	0.433743169	0.436126374	0.438535912
16 .....	0.434782609	0.437158470	0.439560440	0.441988950
16¼ .....	0.438179348	0.440573770	0.442994505	0.445441989
16½ .....	0.441576087	0.443989071	0.446428571	0.448895028
16⅝ .....	0.444972826	0.447404372	0.449862637	0.452348066
16¾ .....	0.448369565	0.450819672	0.453296703	0.455801105
16⅝ .....	0.451766304	0.454234973	0.456730769	0.459254144
16¾ .....	0.455163043	0.457650273	0.460164835	0.462707182
16⅝ .....	0.458559783	0.461065574	0.463598901	0.466160221
17 .....	0.461956522	0.464480874	0.467032967	0.469613260
17¼ .....	0.465353261	0.467896175	0.470467033	0.473066298
17½ .....	0.468750000	0.471311475	0.473901099	0.476519337
17⅝ .....	0.472146739	0.474726776	0.477335165	0.479972376
17¾ .....	0.475543478	0.478142077	0.480769231	0.483245414
17⅝ .....	0.478940217	0.481557377	0.484203297	0.486878453
17¾ .....	0.482336957	0.484972678	0.487637363	0.490331492

TABLE 2—Continued

[Decimal for one day's interest on \$1,000 at various rates of interest, payable semiannually or on a semiannual basis, in regular years of 365 days and in years of 366 days (to determine applicable number of days, see table 1.)]

Rate per annum (percent)	Half-year of 184 days	Half-year of 183 days	Half-year of 182 days	Half-year of 181 days
17⅞ .....	0.485733696	0.488387978	0.491071429	0.493784530
18 .....	0.489130435	0.491803279	0.494505495	0.497237569
18⅛ .....	0.492527174	0.495218579	0.497939560	0.500690608
18¼ .....	0.495923913	0.498633880	0.501373626	0.504143646
18⅜ .....	0.499320652	0.502049180	0.504807692	0.507596685
18½ .....	0.502717391	0.505464481	0.508241758	0.511049724
18⅝ .....	0.506114130	0.508879781	0.511675824	0.514502762
18¾ .....	0.509510870	0.512295082	0.515109890	0.517955801
18⅞ .....	0.512907609	0.515710383	0.518543956	0.521408840
19 .....	0.516304348	0.519125683	0.521978022	0.524861878
19⅛ .....	0.519701087	0.522540984	0.525412088	0.528314917
19¼ .....	0.523097826	0.525956284	0.528846154	0.531767956
19⅜ .....	0.526494565	0.529371585	0.532280220	0.535220994
19½ .....	0.529891304	0.532786885	0.535714286	0.538674033
19⅝ .....	0.533288043	0.536202186	0.539148352	0.542127072
19¾ .....	0.536684783	0.539617486	0.542582418	0.545580110
19⅞ .....	0.540081522	0.543032787	0.546016484	0.549033149
20 .....	0.543478261	0.546448087	0.549450549	0.552486188

3. *Short First Payment Period.* In cases where the first interest payment period for a Treasury non-indexed security covers less than a full half-year period (a “short coupon”), we multiply the daily interest decimal by the number of days from, but not including, the issue date to, and including, the first interest payment date. This calculation results in the amount of the interest payable per \$1,000 par amount. In cases where the par amount of securities is a multiple of \$1,000, we multiply the appropriate multiple by the unrounded interest payment amount per \$1,000 par amount.

#### Example

A 2-year note paying 8⅝% interest was issued on July 2, 1990, with the first interest payment on December 31, 1990. The number of days in the full half-year period of June 30 to December 31, 1990, was 184 (See Table 1.). The number of days for which interest actually accrued was 182 (not including July 2, but including December 31). The daily interest decimal, \$0.227581522 (See Table 2, line for 8⅝%, under the column for half-year of 184 days.), was multiplied by 182, resulting in a payment of \$41.419837004 per \$1,000. For \$20,000 of these notes, \$41.419837004 would be multiplied by 20, resulting in a payment of \$828.39674008 (\$828.40).

4. *Long First Payment Period.* In cases where the first interest payment period for a bond or note covers more than a full half-year period (a “long coupon”), we multiply the daily interest decimal by the number of days from, but not including, the issue date to, and including, the last day of the fractional period that ends one full half-year before the interest payment date. We add that amount to the regular interest amount for the full half-

year ending on the first interest payment date, resulting in the amount of interest payable for \$1,000 par amount. In cases where the par amount of securities is a multiple of \$1,000, the appropriate multiple should be applied to the unrounded interest payment amount per \$1,000 par amount.

#### Example

A 5-year 2-month note paying 7⅞% interest was issued on December 3, 1990, with the first interest payment due on August 15, 1991. Interest for the regular half-year portion of the payment was computed to be \$39.375 per \$1,000 par amount. The fractional portion of the payment, from December 3 to February 15, fell in a 184-day half-year (August 15, 1990, to February 15, 1991). Accordingly, the daily interest decimal for 7⅞% was \$0.213994565. This decimal, multiplied by 74 (the number of days from but not including December 3, 1990, to and including February 15), resulted in interest for the fractional portion of \$15.835597810. When added to \$39.375 (the normal interest payment portion ending on August 15, 1991), this produced a first interest payment of \$55.210597810, or \$55.21 per \$1,000 par amount. For \$7,000 par amount of these notes, \$55.210597810 would be multiplied by 7, resulting in an interest payment of \$386.474184670 (\$386.47).

#### B. Treasury Inflation-Protected Securities

1. *Indexing Process.* We pay interest on marketable Treasury inflation-protected securities on a semiannual basis. We issue inflation-protected securities with a stated rate of interest that remains constant until maturity. Interest payments are based on the security's inflation-adjusted principal at the

time we pay interest. We make this adjustment by multiplying the par amount of the security by the applicable Index Ratio.

2. *Index Ratio.* The numerator of the Index Ratio, the  $\text{Ref CPI}_{\text{Date}}$ , is the index number applicable for a specific day. The denominator of the Index Ratio is the Ref CPI applicable for the original issue date. However, when the dated date is different from the original issue date, the denominator is the Ref CPI applicable for the dated date. The formula for calculating the Index Ratio is:

$$\text{Index Ratio}_{\text{Date}} = \frac{\text{Ref CPI}_{\text{Date}}}{\text{Ref CPI}_{\text{Issue Date}}}$$

Where Date = valuation date

3. *Reference CPI.* The Ref CPI for the first day of any calendar month is the CPI for the

third preceding calendar month. For example, the Ref CPI applicable to April 1 in any year is the CPI for January, which is reported in February. We determine the Ref CPI for any other day of a month by a linear interpolation between the Ref CPI applicable to the first day of the month in which the day falls (in the example, January) and the Ref CPI applicable to the first day of the next month (in the example, February). For interpolation purposes, we truncate calculations with regard to the Ref CPI and the Index Ratio for a specific date to six decimal places, and round to five decimal places.

Therefore the Ref CPI and the Index Ratio for a particular date will be expressed to five decimal places.

(i) The formula for the Ref CPI for a specific date is:

$$\text{Ref CPI}_{\text{Date}} = \text{Ref CPI}_M + \frac{t-1}{D} [\text{Ref CPI}_{M+1} - \text{Ref CPI}_M]$$

Where Date = valuation date

D = the number of days in the month in which Date falls

t = the calendar day corresponding to Date

$\text{CPI}_M$  = CPI reported for the calendar month M by the Bureau of Labor Statistics

$\text{Ref CPI}_M$  = Ref CPI for the first day of the calendar month in which Date falls, e.g.,

$\text{Ref CPI}_{\text{April 1}}$  is the  $\text{CPI}_{\text{January}}$

$\text{Ref CPI}_{M+1}$  = Ref CPI for the first day of the calendar month immediately following Date

(ii) For example, the Ref CPI for April 15, 1996 is calculated as follows:

$$\text{Ref CPI}_{\text{April 15, 1996}} = \text{Ref CPI}_{\text{April 1, 1996}} + \frac{14}{30} [\text{Ref CPI}_{\text{May 1, 1996}} - \text{Ref CPI}_{\text{April 1, 1996}}]$$

where D = 30, t = 15

$\text{Ref CPI}_{\text{April 1, 1996}} = 154.40$ , the non-seasonally adjusted CPI-U for January 1996.

$\text{Ref CPI}_{\text{May 1, 1996}} = 154.90$ , the non-seasonally adjusted CPI-U for February 1996.

(iii) Putting these values in the equation in paragraph (ii) above:

$$\text{Ref CPI}_{\text{April 15, 1996}} = 154.40 + \frac{14}{30} [154.90 - 154.40]$$

$$\text{Ref CPI}_{\text{April 15, 1996}} = 154.63333333$$

This value truncated to six decimals is 154.633333; rounded to five decimals it is 154.63333.

(iv) To calculate the Index Ratio for April 16, 1996, for an inflation-protected security issued on April 15, 1996, the  $\text{Ref CPI}_{\text{April 16, 1996}}$  must first be calculated. Using the same val-

ues in the equation above except that t=16, the  $\text{Ref CPI}_{\text{April 16, 1996}}$  is 154.65000.

The Index Ratio for April 16, 1996 is:

$$\text{Index Ratio}_{\text{April 16, 1996}} = 154.65000/154.63333 = 1.000107803.$$

This value truncated to six decimals is 1.000107; rounded to five decimals it is 1.00011.

4. *Index Contingencies.*

(i) If a previously reported CPI is revised, we will continue to use the previously reported (unrevised) CPI in calculating the principal value and interest payments.

If the CPI is rebased to a different year, we will continue to use the CPI based on the base reference period in effect when the security was first issued, as long as that CPI continues to be published.

(ii) We will replace the CPI with an appropriate alternative index if, while an inflation-protected security is outstanding, the applicable CPI is:

- Discontinued,
- In the judgment of the Secretary, fundamentally altered in a manner materially adverse to the interests of an investor in the security, or
- In the judgment of the Secretary, altered by legislation or Executive Order in a manner materially adverse to the interests of an investor in the security.

(iii) If we decide to substitute an alternative index we will consult with the Bureau of Labor Statistics or any successor agency. We will then notify the public of the substitute index and how we will apply it. Determinations of the Secretary in this regard will be final.

(iv) If the CPI for a particular month is not reported by the last day of the following month, we will announce an index number based on the last available twelve-month change in the CPI. We will base our calculations of our payment obligations that rely on that month's CPI on the index number we announce.

(a) For example, if the CPI for month M is not reported timely, the formula for calculating the index number to be used is:

$$CPI_M = CPI_{M-1} \times \left[ \frac{CPI_{M-1}}{CPI_{M-13}} \right]^{1/12}$$

(b) Generalizing for the last reported CPI issued N months prior to month M:

$$CPI_M = CPI_{M-N} \times \left[ \frac{CPI_{M-N}}{CPI_{M-N-12}} \right]^{N/12}$$

(c) If it is necessary to use these formulas to calculate an index number, we will use that number for all subsequent calculations that rely on the month's index number. We will not replace it with the actual CPI when it is reported, except for use in the above formulas. If it becomes necessary to use the above formulas to derive an index number, we will use the last CPI that has been reported to calculate CPI numbers for months for which the CPI has not been reported timely.

5. *Computation of Interest for a Regular Half-Year Payment Period.* Interest on marketable Treasury inflation-protected securities is

payable on a semiannual basis. The regular interest payment period is a full half-year or six calendar months. Examples of half-year periods are January 15 to July 15, and April 15 to October 15. An interest payment will be a fixed percentage of the value of the inflation-adjusted principal, in current dollars, for the date on which it is paid. We will calculate interest payments by multiplying one-half of the specified annual interest rate for the inflation-protected securities by the inflation-adjusted principal for the interest payment date.

Specifically, we compute a semiannual interest payment on the basis of one-half of one year's interest regardless of the actual number of days in the half-year.

#### Example

A 10-year inflation-protected note paying 3 $\frac{7}{8}$ % interest was issued on January 15, 1999, with the first interest payment on July 15, 1999. The Ref CPI on January 15, 1999 (Ref CPI<sub>IssueDate</sub>) was 164, and the Ref CPI on July 15, 1999 (Ref CPI<sub>Date</sub>) was 166.2. For a par amount of \$100,000, the inflation-adjusted principal on July 15, 1999, was  $(166.2/164) \times \$100,000$ , or \$101,341. This amount was multiplied by .03875/2, or .019375, resulting in a payment of \$1,963.48.

#### C. Treasury Floating Rate Notes

1. *Indexing and Interest Payment Process.* We issue floating rate notes with a daily interest accrual feature. This means that the interest rate "floats" based on changes in the representative index rate. We pay interest on a quarterly basis. The index rate is the High Rate of the 13-week Treasury bill auction announced on the auction results press release that has been converted into a simple-interest money market yield computed on an actual/360 basis and rounded to nine decimal places. Interest payments are based on the floating rate note's variable interest rate from, and including, the dated date or last interest payment date to, but excluding, the next interest payment or maturity date. We make quarterly interest payments by accruing the daily interest amounts and adding those amounts together for the interest payment period.

2. *Interest Rate.* The interest rate on floating rate notes will be the spread plus the index rate (as it may be adjusted on the calendar day following each auction of 13-week bills).

3. *Interest Accrual.* In general, accrued interest for a particular calendar day in an accrual period is calculated by using the index rate from the most recent auction of 13-week bills that took place before the accrual day, plus the spread determined at the time of a new floating rate note auction, divided by 360, subject to a zero-percent minimum daily interest accrual rate. However, a 13-week bill

auction that takes place in the two-business-day period prior to a settlement date or interest payment date will be excluded from the calculation of accrued interest for purposes of the settlement amount or interest payment. Any changes in the index rate that would otherwise have occurred during this two-business-day period will occur on the first calendar day following the end of the period.

#### 4. *Index Contingencies.*

(i) If Treasury were to discontinue auctions of 13-week bills, the Secretary has authority to determine and announce a new index for outstanding floating rate notes.

(ii) If Treasury were to not conduct a 13-week bill auction in a particular week, then the interest rate in effect for the notes at the time of the last 13-week bill auction results announcement will remain in effect until such time, if any, as the results of a 13-week Treasury auction are again announced by Treasury. Treasury reserves the right to change the index rate for any newly issued floating rate note.

#### D. *Accrued Interest*

1. You will have to pay accrued interest on a Treasury bond or note when interest accrues prior to the issue date of the security. Because you receive a full interest payment despite having held the security for only a portion of the interest payment period, you must compensate us through the payment of accrued interest at settlement.

2. For a Treasury non-indexed security, if accrued interest covers a fractional portion of a full half-year period, the number of days in the full half-year period and the stated interest rate will determine the daily interest decimal to use in computing the accrued interest. We multiply the decimal by the number of days for which interest has accrued.

3. If a reopened bond or note has a long first interest payment period (a "long coupon"), and the dated date for the reopened issue is less than six full months before the first interest payment, the accrued interest will fall into two separate half-year periods. A separate daily interest decimal must be multiplied by the respective number of days in each half-year period during which interest has accrued.

4. We round all accrued interest computations to five decimal places for a \$1,000 par amount, using normal rounding procedures. We calculate accrued interest for a par amount of securities greater than \$1,000 by applying the appropriate multiple to accrued interest payable for a \$1,000 par amount, rounded to five decimal places. We calculate accrued interest for a par amount of securities less than \$1,000 by applying the appropriate fraction to accrued interest payable for a \$1,000 par amount, rounded to five decimal places.

5. For an inflation-protected security, we calculate accrued interest as shown in section III, paragraphs A and B of this appendix.

*Examples—(1) Treasury Non-indexed Securities—(i) Involving One Half-Year:* A note paying interest at a rate of 6¾%, originally issued on May 15, 2000, as a 5-year note with a first interest payment date of November 15, 2000, was reopened as a 4-year 9-month note on August 15, 2000. Interest had accrued for 92 days, from May 15 to August 15. The regular interest period from May 15 to November 15, 2000, covered 184 days. Accordingly, the daily interest decimal, \$0.183423913, multiplied by 92, resulted in accrued interest payable of \$16.874999996, or \$16.87500, for each \$1,000 note purchased. If the notes have a par amount of \$150,000, then 150 is multiplied by \$16.87500, resulting in an amount payable of \$2,531.25.

#### (2) *Involving Two Half-Years:*

A 10¾% bond, originally issued on July 2, 1985, as a 20-year 1-month bond, with a first interest payment date of February 15, 1986, was reopened as a 19-year 10-month bond on November 4, 1985. Interest had accrued for 44 days, from July 2 to August 15, 1985, during a 181-day half-year (February 15 to August 15); and for 81 days, from August 15 to November 4, during a 184-day half-year (August 15, 1985, to February 15, 1986). Accordingly, \$0.296961326 was multiplied by 44, and \$0.292119565 was multiplied by 81, resulting in products of \$13.066298344 and \$23.661684765 which, added together, resulted in accrued interest payable of \$36.727983109, or \$36.72798, for each \$1,000 bond purchased. If the bonds have a par amount of \$11,000, then 11 is multiplied by \$36.72798, resulting in an amount payable of \$404.00778 (\$404.01).

6. For a floating rate note, if accrued interest covers a portion of a full quarterly interest payment period, we calculate accrued interest as shown in section IV, paragraphs C and D of this appendix.

## II. FORMULAS FOR CONVERSION OF NON-INDEXED SECURITY YIELDS TO EQUIVALENT PRICES

### *Definitions*

P = price per 100 (dollars), rounded to six places, using normal rounding procedures.

C = the regular annual interest per \$100, payable semiannually, e.g., 6.125 (the decimal equivalent of a 6¼% interest rate).

i = nominal annual rate of return or yield to maturity, based on semiannual interest payments and expressed in decimals, e.g., .0719.

n = number of full semiannual periods from the issue date to maturity, except that, if the issue date is a coupon frequency date, n will be one less than the number of full semiannual periods remaining to maturity. Coupon frequency dates are the two semiannual dates based on the maturity date of



each note or bond issue. For example, a security maturing on November 15, 2015, would have coupon frequency dates of May 15 and November 15.

$r$  = (1) number of days from the issue date to the first interest payment (regular or short first payment period), or (2) number of days in fractional portion (or "initial short period") of long first payment period.

$s$  = (1) number of days in the full semiannual period ending on the first interest payment date (regular or short first payment period), or (2) number of days in the full semiannual period in which the fractional portion of a long first payment period falls, ending at the onset of the regular portion of the first interest payment.

$v^n = 1 / [1 + (i/2)]^n$  = present value of 1 due at the end of  $n$  periods.

$a_n = (1 - v^n) / (i/2) = v + v^2 + v^3 + \dots + v^n$  = present value of 1 per period for  $n$  periods

*Special Case:* If  $i = 0$ , then  $a_n = n$ . Furthermore, when  $i = 0$ ,  $a_n$  cannot be calculated using the formula:  $(1 - v^n)/(i/2)$ . In the special case where  $i = 0$ ,  $a_n$  must be calculated as the summation of the individual present values (i.e.,  $v + v^2 + v^3 + \dots + v^n$ ). Using the summation method will always confirm that  $a_n = n$  when  $i = 0$ .

$A$  = accrued interest.

*A. For non-indexed securities with a regular first interest payment period:*

Formula:

$$P[1 + (r/s)(i/2)] = (C/2)(r/s) + (C/2)a_n + 100v^n.$$

Example:

For an 8¾% 30-year bond issued May 15, 1990, due May 15, 2020, with interest payments on November 15 and May 15, solve for the price per 100 ( $P$ ) at a yield of 8.84%.

Definitions: G12752

$C = 8.75$ .

$i = .0884$ .

$r = 184$  (May 15 to November 15, 1990).

$s = 184$  (May 15 to November 15, 1990).

$n = 59$  (There are 60 full semiannual periods, but  $n$  is reduced by 1 because the issue date is a coupon frequency date.)

$v^n = 1 / [(1 + .0884/2)]^{59}$ , or .0779403508.

$a_n = (1 - .0779403508) / .0442$ , or 20.8610780353.

Resolution:

$$P[1 + (r/s)(i/2)] = (C/2)(r/s) + (C/2)a_n + 100v^n$$

or

$$P[1 + (184/184)(.0884/2)] = (8.75/2)(184/184) + (8.75/2)(20.8610780353) + 100(.0779403508).$$

$$(1) P[1 + .0442] = 4.375 + 91.2672164044 + 7.7940350840.$$

$$(2) P[1.0442] = 103.4362514884.$$

$$(3) P = 103.4362514884 / 1.0442.$$

$$(4) P = 99.057893.$$

*B. For non-indexed securities with a short first interest payment period:*

Formula:

$$P[1 + (r/s)(i/2)] = (C/2)(r/s) + (C/2)a_n + 100v^n.$$

Example:

For an 8½% 2-year note issued April 2, 1990, due March 31, 1992, with interest payments on September 30 and March 31, solve for the price per 100 ( $P$ ) at a yield of 8.59%.

Definitions:

$C = 8.50$ .

$i = .0859$ .

$n = 3$ .

$r = 181$  (April 2 to September 30, 1990).

$s = 183$  (March 31 to September 30, 1990).

$v^n = 1 / [(1 + .0859/2)]^3$ , or .8814740565.

$a_n = (1 - .8814740565) / .04295$ , or 2.7596261590.

Resolution:

$$P[1 + (r/s)(i/2)] = (C/2)(r/s) + (C/2)a_n + 100v^n$$

or

$$P[1 + (181/183)(.0859/2)] = (8.50/2)(181/183) + (8.50/2)(2.7596261590) + 100(.8814740565).$$

$$(1) P[1 + .042480601] = 4.2035519126 + 11.7284111757 + 88.14740565.$$

$$(2) P[1.042480601] = 104.0793687354.$$

$$(3) P = 104.0793687354 / 1.042480601.$$

$$(4) P = 99.838183.$$

*C. For non-indexed securities with a long first interest payment period:*

Formula:

$$P[1 + (r/s)(i/2)] = [(C/2)(r/s)]v + (C/2)a_n + 100v^n.$$

Example:

For an 8½% 5-year 2-month note issued March 1, 1990, due May 15, 1995, with interest payments on November 15 and May 15 (first payment on November 15, 1990), solve for the price per 100 ( $P$ ) at a yield of 8.53%.

Definitions:

$C = 8.50$ .

$i = .0853$ .

$n = 10$ .

$r = 75$  (March 1 to May 15, 1990, which is the fractional portion of the first interest payment).

$s = 181$  (November 15, 1989, to May 15, 1990).

$v = 1 / (1 + .0853/2)$ , or .9590946147.

$v^n = 1 / (1 + .0853/2)^{10}$ , or .658589

$a_n = (1 - .658589) / .04265$ , or 8.0049454082.

Resolution:

$$P[1 + (r/s)(i/2)] = [(C/2)(r/s)]v + (C/2)a_n + 100v^n$$

or

$$P[1 + (75/181)(.0853/2)] = [(8.50/2)(75/181)](.9590946147) + (8.50/2)(8.0049454082) + 100(.6585890783).$$

$$(1) P[1 + .017672652] = 1.6890133062 + 34.0210179850 + 65.8589078339.$$

$$(2) P[1.017672652] = 101.5689391251.$$

$$(3) P = 101.5689391251 / 1.017672652.$$

$$(4) P = 99.805118.$$

*D. (1) For non-indexed securities reopened during a regular interest period where the purchase price includes predetermined accrued interest.*

*(2) For new non-indexed securities accruing interest from the coupon frequency date immediately preceding the issue date, with the interest rate established in the auction being used to*

## Fiscal Service, Treasury

## Pt. 356, App. B

determine the accrued interest payable on the issue date.

Formula:

$$(P + A)[1 + (r/s)(i/2)] = C/2 + (C/2)a_n + 100v^n.$$

Where:

$$A = [(s - r)/s](C/2).$$

Example:

For a 9½% 10-year note with interest accruing from November 15, 1985, issued November 29, 1985, due November 15, 1995, and interest payments on May 15 and November 15, solve for the price per 100 (P) at a yield of 9.54%. Accrued interest is from November 15 to November 29 (14 days).

Definitions:

$$C = 9.50.$$

$$i = .0954.$$

$$n = 19.$$

$$r = 167 \text{ (November 29, 1985, to May 15, 1986).}$$

$$s = 181 \text{ (November 15, 1985, to May 15, 1986).}$$

$$v^n = 1 / [(1 + .0954/2)]^{19}, \text{ or } .4125703996.$$

$$a_n = (1 - .4125703996) / .0477, \text{ or } 12.3150859630.$$

$$A = [(181 - 167) / 181](9.50/2), \text{ or } .367403.$$

Resolution:

$$(P + A)[1 + (r/s)(i/2)] = C/2 + (C/2)a_n + 100v^n \text{ or } (P + .367403)[1 + (167/181)(.0954/2)] = (9.50/2) + (9.50/2)(12.3150859630) + 100(.4125703996).$$

$$(1) (P + .367403)[1 + .044010497] = 4.75 + 58.4966583243 + 41.25703996.$$

$$(2) (P + .367403)[1.044010497] = 104.5036982843.$$

$$(3) (P + .367403) = 104.5036982843 / 1.044010497.$$

$$(4) (P + .367403) = 100.098321.$$

$$(5) P = 100.098321 - .367403.$$

$$(6) P = 99.730918.$$

E. For non-indexed securities reopened during the regular portion of a long first payment period:

Formula:

$$(P + A)[1 + (r/s)(i/2)] = (r's'')(C/2) + C/2 + (C/2)a_n + 100v^n.$$

Where:

$$A = AI' + AI,$$

$$AI' = (r's'')(C/2),$$

$$AI = [(s - r) / s](C/2), \text{ and}$$

r = number of days from the reopening date to the first interest payment date,

s = number of days in the semiannual period for the regular portion of the first interest payment period,

r' = number of days in the fractional portion (or "initial short period") of the first interest payment period,

s'' = number of days in the semiannual period ending with the commencement date of the regular portion of the first interest payment period.

Example:

A 10¾% 19-year 9-month bond due August 15, 2005, is issued on July 2, 1985, and reopened on November 4, 1985, with interest payments on February 15 and August 15 (first payment on February 15, 1986), solve for the price per 100 (P) at a yield of 10.47%. Accrued

interest is calculated from July 2 to November 4.

Definitions:

$$C = 10.75.$$

$$i = .1047.$$

$$n = 39.$$

$$r = 103 \text{ (November 4, 1985, to February 15, 1986).}$$

$$s = 184 \text{ (August 15, 1985, to February 15, 1986).}$$

$$r' = 44 \text{ (July 2 to August 15, 1985).}$$

$$s'' = 181 \text{ (February 15 to August 15, 1985).}$$

$$v^n = 1 / [(1 + .1047 / 2)]^{39}, \text{ or } .1366947986.$$

$$a_n = (1 - .1366947986) / .05235, \text{ or } 16.4910258142.$$

$$AI' = (44 / 181)(10.75 / 2), \text{ or } 1.306630.$$

$$AI = [(184 - 103) / 184](10.75 / 2), \text{ or } 2.366168.$$

$$A = AI' + AI, \text{ or } 3.672798.$$

Resolution:

$$(P + A)[1 + (r/s)(i/2)] = (r's'')(C/2) + C/2 + (C/2)a_n + 100v^n \text{ or}$$

$$(P + 3.672798)[1 + (103/184)(.1047/2)] = (44/181)(10.75/2) + 10.75/2 + (10.75/2)(16.4910258142) + 100(.1366947986).$$

$$(1) (P + 3.672798)[1 + .02930462] = 1.3066298343 + 5.375 + 88.6392637512 + 13.6694798628.$$

$$(2) (P + 3.672798)[1.02930462] = 108.9903734482.$$

$$(3) (P + 3.672798) = 108.9903734482 / 1.02930462.$$

$$(4) (P + 3.672798) = 105.887384.$$

$$(5) P = 105.887384 - 3.672798.$$

$$(6) P = 102.214586.$$

F. For non-indexed securities reopened during a short first payment period:

Formula:

$$(P + A)[1 + (r/s)(i/2)] = (r's'')(C/2) + (C/2)a_n + 100v^n.$$

Where:

$$A = [(r' - r)/s](C/2) \text{ and}$$

r' = number of days from the original issue date to the first interest payment date.

Example:

For a 10½% 8-year note due May 15, 1991, originally issued on May 16, 1983, and reopened on August 15, 1983, with interest payments on November 15 and May 15 (first payment on November 15, 1983), solve for the price per 100 (P) at a yield of 10.53%. Accrued interest is calculated from May 16 to August 15.

Definitions:

$$C = 10.50.$$

$$i = .1053.$$

$$n = 15.$$

$$r = 92 \text{ (August 15, 1983, to November 15, 1983).}$$

$$s = 184 \text{ (May 15, 1983, to November 15, 1983).}$$

$$r' = 183 \text{ (May 16, 1983, to November 15, 1983).}$$

$$v^n = 1 / [(1 + .1053/2)]^{15}, \text{ or } .4631696332.$$

$$a_n = (1 - .4631696332) / .05265, \text{ or } 10.1962082956.$$

$$A = [(183 - 92) / 184](10.50 / 2), \text{ or } 2.596467.$$

Resolution:

$$(P + A)[1 + (r/s)(i/2)] = (r's'')(C/2) + (C/2)a_n + 100v^n \text{ or}$$

$$(P + 2.596467)[1 + (92/184)(.1053/2)] = (183/184)(10.50/2) + (10.50/2)(10.1962082956) + 100(.4631696332).$$

**Pt. 356, App. B**

**31 CFR Ch. II (7-1-14 Edition)**

- (1)  $(P + 2.596467)[1 + .026325] = 5.2214673913 + 53.5300935520 + 46.31696332$ .
- (2)  $(P + 2.596467)[1.026325] = 105.0685242633$ .
- (3)  $(P + 2.596467) = 105.0685242633 / 1.026325$ .
- (4)  $(P + 2.596467) = 102.373541$ .
- (5)  $P = 102.373541 - 2.596467$ .
- (6)  $P = 99.777074$ .

G. *For non-indexed securities reopened during the fractional portion (initial short period) of a long first payment period:*

Formula:

$$(P + A)[1 + (r/s)(i/2)] = [(r'/s)(C/2)]v + (C/2)a_n + 100v^n$$

Where:

$A = [(r' - r)/s](C/2)$ , and

$r$  = number of days from the reopening date to the end of the short period.

$r'$  = number of days in the short period.

$s$  = number of days in the semiannual period ending with the end of the short period.

Example:

For a 9% 6-year 2-month note due December 15, 1994, originally issued on October 15, 1988, and reopened on November 15, 1988, with interest payments on June 15 and December 15 (first payment on June 15, 1989), solve for the price per 100 ( $P$ ) at a yield of 9.79%. Accrued interest is calculated from October 15 to November 15.

Definitions:

$C = 9.75$ .

$i = .0979$ .

$n = 12$ .

$r = 30$  (November 15, 1988, to December 15, 1988).

$s = 183$  (June 15, 1988, to December 15, 1988).

$r' = 61$  (October 15, 1988, to December 15, 1988).

$v = 1 / (1 + .0979/2)$ , or .9533342867.

$v^n = [1 / (1 + .0979/2)]^{12}$ , or .5635631040.

$a_n = (1 - .5635631040) / (.04895)$ , or 8.9159733613.

$A = [(61 - 30)/183](9.75/2)$ , or .825820.

Resolution:

$$(P + A)[1 + (r/s)(i/2)] = [(r'/s)(C/2)]v + (C/2)a_n + 100v^n \text{ or}$$

$$(P + .825820)[1 + (30/183)(.0979/2)] = [(61/183)(9.75/2)](.9533342867) + (9.75/2)(8.9159733613) + 100(.5635631040).$$

$$(1) (P + .825820)[1 + .00802459] = 1.549168216 + 43.4653701362 + 56.35631040.$$

$$(2) (P + .825820)[1.00802459] = 101.3708487520.$$

$$(3) (P + .825820) = 101.3708487520 / 1.00802459.$$

$$(4) (P + .825820) = 100.563865.$$

$$(5) P = 100.563865 - .825820.$$

$$(6) P = 99.738045.$$

### III. FORMULAS FOR CONVERSION OF INFLATION-INDEXED SECURITY YIELDS TO EQUIVALENT PRICES

#### Definitions

$P$  = unadjusted or real price per 100 (dollars).  
 $P_{\text{adj}}$  = inflation adjusted price;  $P \times \text{Index Ratio}_{\text{Date}}$ .

$A$  = unadjusted accrued interest per \$100 original principal.

$A_{\text{adj}}$  = inflation adjusted accrued interest;  $A \times \text{Index Ratio}_{\text{Date}}$ .

$SA$  = settlement amount including accrued interest in current dollars per \$100 original principal;  $P_{\text{adj}} + A_{\text{adj}}$ .

$r$  = days from settlement date to next coupon date.

$s$  = days in current semiannual period.

$i$  = real yield, expressed in decimals (e.g., 0.0325).

$C$  = real annual coupon, payable semiannually, in terms of real dollars paid on \$100 initial, or real, principal of the security.

$n$  = number of full semiannual periods from issue date to maturity date, except that, if the issue date is a coupon frequency date,  $n$  will be one less than the number of full semiannual periods remaining until maturity. Coupon frequency dates are the two semiannual dates based on the maturity date of each note or bond issue. For example, a security maturing on July 15, 2026 would have coupon frequency dates of January 15 and July 15.

$v^n = 1/(1 + i/2)^n$  = present value of 1 due at the end of  $n$  periods.

$a_n = (1 - v^n) / (i/2) = v + v^2 + v^3 + \dots + v^n$  = present value of 1 per period for  $n$  periods.

Special Case: If  $i = 0$ , then  $a_n = n$ . Furthermore, when  $i = 0$ ,  $a_n$  cannot be calculated using the formula:  $(1 - v^n)/(i/2)$ . In the special case where  $i = 0$ ,  $a_n$  must be calculated as the summation of the individual present values (i.e.,  $v + v^2 + v^3 + \dots + v^n$ ). Using the summation method will always confirm that  $a_n = n$  when  $i = 0$ .

Date = valuation date.

$D$  = the number of days in the month in which Date falls.

$t$  = calendar day corresponding to Date.

$CPI$  = Consumer Price Index number.

$CPI_M$  = CPI reported for the calendar month  $M$  by the Bureau of Labor Statistics.

$\text{Ref } CPI_M$  = reference CPI for the first day of the calendar month in which Date falls (also equal to the CPI for the third preceding calendar month), e.g.,  $\text{Ref } CPI_{\text{April } 1}$  is the  $CPI_{\text{January}}$ .

$\text{Ref } CPI_{M+1}$  = reference CPI for the first day of the calendar month immediately following Date.

$\text{Ref } CPI_{\text{Date}} = \text{Ref } CPI_M - [(t - 1)/D][\text{Ref } CPI_{M+1} - \text{Ref } CPI_M]$ .

$\text{Index Ratio}_{\text{Date}} = \text{Ref } CPI_{\text{Date}} / \text{Ref } CPI_{\text{IssueDate}}$ .

NOTE: When the Issue Date is different from the Dated Date, the denominator is the  $\text{Ref } CPI_{\text{DatedDate}}$ .

A. *For inflation-indexed securities with a regular first interest payment period:*  
 Formulas:

$$P = \frac{(C/2) + (C/2)a_n + 100v^n}{1 + (r/s)(i/2)} - [(s-r)/s](C/2)$$

$$P_{\text{adj}} = P \times \text{Index Ratio}_{\text{Date.}}$$

$$A = [(s-r)/s] \times (C/2).$$

$$A_{\text{adj}} = A \times \text{Index Ratio}_{\text{Date.}}$$

$$SA = P_{\text{adj}} + A_{\text{adj}}$$

$$\text{Index Ratio}_{\text{Date}} = \text{Ref CPI}_{\text{Date}} / \text{Ref CPI}_{\text{IssueDate.}}$$

Example:

We issued a 10-year inflation-indexed note on January 15, 1999. The note was issued at a discount to yield of 3.898% (real). The note bears a 3% real coupon, payable on July 15 and January 15 of each year. The base CPI index applicable to this note is 164. (We normally derive this number using the interpolative process described in appendix B, section I, paragraph B.)

$$P = \frac{(C/2) + (C/2)a_n + 100v^n}{1 + (r/s)(i/2)} - [(s-r)/s](C/2)$$

$$P = \frac{(3.875/2) + (3.875/2)(15.752459107) + 100(0.692984572)}{1 + (181/181)(0.03898/2)} - [(181 - 181)/181](3.875/2)$$

$$P = \frac{1.9375 + 30.52038952 + 69.29845720}{1.01949000} - 0$$

$$P = \frac{101.75634672}{1.01949000}$$

$$P = 99.811030.$$

$$P_{\text{adj}} = P \times \text{Index Ratio}_{\text{Date.}}$$

$$P_{\text{adj}} = 99.811030 \times 1 = 99.811030.$$

$$SA = P_{\text{adj}} \times A_{\text{adj.}}$$

$$SA = 99.811030 + 0 = 99.811030.$$

NOTE: For the real price (P), we have rounded to six places. These amounts are based on 100 par value.

B. (1) *For inflation-indexed securities reopened during a regular interest period where the purchase price includes predetermined accrued interest.*

(2) *For new inflation-indexed securities accruing interest from the coupon frequency date immediately preceding the issue date, with the interest rate established in the auction being used to determine the accrued interest payable on the issue date.*

**Bidding:** The dollar amount of each bid is in terms of the par amount. For example, if the Ref CPI applicable to the issue date of the note is 120, and the reference CPI applicable to the reopening issue date is 132, a bid of \$10,000 will in effect be a bid of \$10,000 × (132/120), or \$11,000.

Formulas:

$$P = \frac{(C/2) + (C/2)a_n + 100v^n}{1 + (r/s)(i/2)} - [(s-r)/s](C/2)$$

$$P_{\text{adj}} = P \times \text{Index Ratio}_{\text{Date.}}$$

$$A = [(s-r)/s] \times (C/2).$$

$$A_{\text{adj}} = A \times \text{Index Ratio}_{\text{Date.}}$$

Definitions:

$$C = 3.875.$$

$$i = 0.03898.$$

$$n = 19 \text{ (There are 20 full semiannual periods but } n \text{ is reduced by 1 because the issue date is a coupon frequency date.)}$$

$$r = 181 \text{ (January 15, 1999 to July 15, 1999).}$$

$$s = 181 \text{ (January 15, 1999 to July 15, 1999).}$$

$$\text{Ref CPI}_{\text{Date}} = 164.$$

$$\text{Ref CPI}_{\text{IssueDate}} = 164.$$

Resolution:

$$\text{Index Ratio}_{\text{Date}} = \text{Ref CPI}_{\text{Date}} / \text{Ref CPI}_{\text{IssueDate}} = 164/164 = 1.$$

$$A = [(181 - 181)/181] \times 3.875/2 = 0.$$

$$A_{\text{adj}} = 0 \times 1 = 0.$$

$$v^n = 1/(1 + i/2)^n = 1/(1 + .03898/2)^{19} = 0.692984572.$$

$$a_n = (1 - v^n)/(i/2) = (1 - 0.692984572) / (.03898/2) = 15.752459107.$$

Formula:

$$SA = P_{\text{adj}} + A_{\text{adj.}}$$

$$\text{Index Ratio}_{\text{Date}} = \text{Ref CPI}_{\text{Date}} / \text{Ref CPI}_{\text{IssueDate.}}$$

Example:

We issued a 3% 10-year inflation-indexed note on January 15, 1998, with interest payments on July 15 and January 15. For a reopening on October 15, 1998, with inflation compensation accruing from January 15, 1998 to October 15, 1998, and accrued interest accruing from July 15, 1998 to October 15, 1998 (92 days), solve for the price per 100 (P) at a real yield, as determined in the reopening auction, of 3.65%. The base index applicable to the issue date of this note is 161.55484 and the reference CPI applicable to October 15, 1998, is 163.29032.

Definitions:

$$C = 3.625.$$

$$i = 0.0365.$$

$$n = 18.$$

$$r = 92 \text{ (October 15, 1998 to January 15, 1999).}$$

$$s = 184 \text{ (July 15, 1998 to January 15, 1999).}$$

$$\text{Ref CPI}_{\text{Date}} = 163.29032.$$

$$\text{Ref CPI}_{\text{IssueDate}} = 161.55484.$$

Resolution:

$$\text{Index Ratio}_{\text{Date}} = \text{Ref CPI}_{\text{Date}} / \text{Ref CPI}_{\text{IssueDate}} = 163.29032/161.55484 = 1.01074.$$

$$v^n = 1/(1 + i/2)^n = 1/(1 + .0365/2)^{18} = 0.722138438.$$

$$a_n = (1 - v^n)/(i/2) = (1 - 0.722138438) / (.0365/2) = 15.225291068.$$

Formula:

$$P = \frac{(C/2) + (C/2)a_n + 100v^n}{1 + (r/s)(i/2)} - [(s - r)/s](C/2)$$

$$P = \frac{(3.625/2) + (3.625/2)(15.225291068) + 100(0.722138438)}{1 + (92/184)(0.0365/2)} - [(184 - 92)/184](3.625/2)$$

$$P = \frac{1.8125 + 27.59584006 + 72.21384380}{1.009125} - (92/184)(1.8125)$$

$$P = \frac{101.62218386}{1.009125} - 0.906250$$

$P = 100.703267 - 0.906250.$   
 $P = 99.797017.$   
 $P_{\text{adj}} = P \times \text{Index Ratio}_{\text{Date}}.$   
 $P_{\text{adj}} = 99.797017 \times 1.01074 = 100.86883696.$   
 $P_{\text{adj}} = 100.868837.$   
 $A = [(184 - 92)/184] \times 3.625/2 = 0.906250.$   
 $A_{\text{adj}} = A \times \text{Index Ratio}_{\text{Date}}.$   
 $A_{\text{adj}} = 0.906250 \times 1.01074 = 0.91598313.$   
 $A_{\text{adj}} = 0.915983.$   
 $SA = P_{\text{adj}} + A_{\text{adj}} = 100.868837 + 0.915983.$   
 $SA = 101.784820.$

NOTE: For the real price (P), and the inflation-adjusted price ( $P_{\text{adj}}$ ), we have rounded to six places. For accrued interest (A) and the adjusted accrued interest ( $A_{\text{adj}}$ ), we have rounded to six places. These amounts are based on 100 par value.

#### IV. FORMULAS FOR CONVERSION OF FLOATING RATE NOTE DISCOUNT MARGINS TO EQUIVALENT PRICES

##### *Definitions for Newly Issued Floating Rate Notes*

P = the price per \$100 par value.

$T_0$  = the issue date.  
 $N$  = the total number of quarterly interest payments.  
 $i$  and  $k$  = indexes that identify the sequence of interest payment dates.  
 $T_i$  = the  $i^{\text{th}}$  quarterly interest payment date.  
 $T_i - T_{i-1}$  = the number of days between the interest payment date  $T_i$  and the preceding interest payment date.  
 $T_N$  = the maturity date.  
 $r$  = the index rate applicable to the issue date.  
 $s$  = the spread.  
 $m$  = the discount margin.

A. For newly issued floating rate notes issued at par:

Formula:

$$P = \sum_{i=1}^N \left( \frac{100 \times \frac{1}{360} (T_i - T_{i-1}) \times \max(r + s, 0)}{\prod_{k=1}^i \left( 1 + \frac{1}{360} (T_k - T_{k-1}) \times (r + m) \right)} \right) + \frac{100}{\prod_{k=1}^N \left( 1 + \frac{1}{360} (T_k - T_{k-1}) \times (r + m) \right)}$$

Example:

The purpose of this example is to demonstrate how a floating rate note price is derived at the time of original issuance. Additionally, this example depicts the association of the July 31, 2012 issue date and the two-business-day lockout period. For a new two-year floating rate note auctioned on July 25, 2012, and issued on July 31, 2012, with a maturity date of July 31, 2014, and an interest accrual rate on the issue date of 0.215022819% (index rate of 0.095022819% plus a

spread of 0.120%), solve for the price per 100 (P). This interest accrual rate is used for each daily interest accrual over the life of the security for the purposes of this example. In a new issuance (not a reopening) of a floating rate note, the discount margin determined at auction will be equal to the spread.

Definitions:

$T_0$  = July 31, 2012.

$N$  = 8.

$T_N$  = July 31, 2014.

**Fiscal Service, Treasury**

**Pt. 356, App. B**

$r = 0.095022819\%$ .  
 $s = 0.120\%$ .  
 $m = 0.120\%$ .

As of the issue date the latest 13-week bill, auctioned at least two days prior, has the following information:

**TABLE 1—13-WEEK BILL AUCTION DATA**

Auction date	Issue date	Maturity date	Auction clearing price	Auction high rate	Index rate
7/23/2012	7/26/2012	10/25/2012	99.975986	0.095%	0.095022819%

The rationale for using a 13-week bill auction that has occurred at least two days prior to the issue date is due to the two-business-day lockout period. This lockout period applies only to the issue date and interest payment dates, thus any 13-week bill auction

that occurs during the two-day lockout period is not used for calculations related to the issue date and interest payment dates. The following sample calendar depicts this relationship for the floating rate note issue date.

**July 2012**

Sunday 22nd	<b>Monday 23<sup>rd</sup> 13-week bill auction</b>	Tuesday 24 <sup>th</sup>	<b>Wednesday 25<sup>th</sup> Auction date</b>	Thursday 26 <sup>th</sup>	<b>Friday 27<sup>th</sup> Lockout Day 1</b>	Saturday 28 <sup>th</sup>
Sunday 29 <sup>th</sup>	<b>Monday 30<sup>th</sup> Lockout Day 2 13-week bill auction (not applicable for July 31 calculations)</b>	<b>Tuesday 31<sup>st</sup> Issue date</b>				

*Computing the index rate*

The index rate that equals the simple-interest money market yield on an actual/360 basis is computed as follows:

$$r = \frac{D}{1 - \frac{\Delta T}{360} D}$$

where  $D$  is the discount rate (or auction high rate), and  $\Delta T$  represents the number of days from (and including) the issue date of the 13-week bill to (but excluding) the maturity

date of the 13-week bill. In the table above,  $r = \frac{0.095\%}{1 - \frac{91}{360} \times 0.095\%} = 0.095022819\%$ .

*Computing the Projected Cash Flows*

The following table presents the future interest payment dates and the number of days between them.

TABLE 2—PAYMENT DATES

Dates	Days between dates
Issue Date: $T_0 = 7/31/2012$	
1st Interest Date: $T_1 = 10/31/2012$	$T_1 - T_0 = 92$
2nd Interest Date: $T_2 = 1/31/2013$	$T_2 - T_1 = 92$
3rd Interest Date: $T_3 = 4/30/2013$	$T_3 - T_2 = 89$
4th Interest Date: $T_4 = 7/31/2013$	$T_4 - T_3 = 92$
5th Interest Date: $T_5 = 10/31/2013$	$T_5 - T_4 = 92$
6th Interest Date: $T_6 = 1/31/2014$	$T_6 - T_5 = 92$
7th Interest Date: $T_7 = 4/30/2014$	$T_7 - T_6 = 89$
8th Interest & Maturity Dates: $T_8 = 7/31/2014$	$T_8 - T_7 = 92$

Let

$$a_i = 100 \times \max(r + s, 0)/360$$

and

$$A_i = a_i \times (T_i - T_{i-1}) + 100 \times 1_{\{i=8\}}$$

$a_i$  represents the daily projected interest, for a \$100 par value, that will accrue between the future interest payment dates  $T_{i-1}$  and  $T_i$ , where  $i = 1, 2, \dots, 8$ .  $a_i$ 's are computed using the spread  $s = 0.120\%$  obtained at the auction, and the fixed index rate of  $r = 0.095022819\%$  applicable to the issue date (7/31/2012). For example:

$$a_1 = 100 \times \max(0.00095022819 + 0.00120, 0)/360 = 0.000597286$$

$A_i$  represents the projected cash flow the floating rate note holder will receive, for a \$100 par value, at the future interest payment date  $T_i$ , where  $i = 1, 2, \dots, 8$ .  $T_i - T_{i-1}$  is the number of days between the future interest payment dates  $T_{i-1}$  and  $T_i$ . To account for the payback of the par value, the variable  $1_{\{i=8\}}$  takes the value 1 if the payment date is

the maturity date, or 0 otherwise. For example:

$$A_1 = 92 \times 0.000597286 = 0.054950312$$

and

$$A_8 = 92 \times 0.000597286 + 100 = 100.054950312$$

Let

$$B_i = 1 + (r + m) \times (T_i - T_{i-1})/360$$

$B_i$  represents the projected compound factor between the future dates  $T_{i-1}$  and  $T_i$ , where  $i = 1, 2, \dots, 8$ . All  $B_i$ 's are computed using the discount margin  $m = 0.120\%$  (equals the spread determined at the auction), and the fixed index rate of  $r = 0.095022819\%$  applicable to the issue date (7/31/2012). For example:

$$B_3 = 1 + (0.00095022819 + 0.00120) \times 89/360 = 1.000531584.$$

The following table shows the projected daily accrued interest values for \$100 par value ( $a_i$ 's), cash flows at interest payment dates ( $A_i$ 's), and the compound factors between payment dates ( $B_i$ 's).

TABLE 3—PROJECTED CASH FLOWS AND COMPOUND FACTORS

$i$	$a_i$	$A_i$	$B_i$
1	0.000597286	0.054950312	1.000549503
2	0.000597286	0.054950312	1.000549503
3	0.000597286	0.053158454	1.000531584
4	0.000597286	0.054950312	1.000549503
5	0.000597286	0.054950312	1.000549503
6	0.000597286	0.054950312	1.000549503
7	0.000597286	0.053158454	1.000531584
8	0.000597286	100.054950312	1.000549503

#### Computing the Price

The price is computed as follows:

$$P = \left[ \frac{A_1}{B_1} + \frac{A_2}{B_1 B_2} + \frac{A_3}{B_1 B_2 B_3} + \frac{A_4}{B_1 B_2 B_3 B_4} + \frac{A_5}{B_1 B_2 B_3 B_4 B_5} + \right. \\ \left. \frac{A_6}{B_1 B_2 B_3 B_4 B_5 B_6} + \frac{A_7}{B_1 B_2 B_3 B_4 B_5 B_6 B_7} + \frac{A_8}{B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8} \right]$$

$$P = \left[ \frac{0.054950312}{B_1} + \frac{0.054950312}{B_1 B_2} + \frac{0.053158454}{B_1 B_2 B_3} + \frac{0.054950312}{B_1 B_2 B_3 B_4} + \right. \\ \left. \frac{0.054950312}{B_1 B_2 B_3 B_4 B_5} + \frac{0.054950312}{B_1 B_2 B_3 B_4 B_5 B_6} + \frac{0.053158454}{B_1 B_2 B_3 B_4 B_5 B_6 B_7} + \frac{100.054950312}{B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8} \right]$$

$$P = \left[ \frac{0.054950312}{1.000549503} + \frac{0.054950312}{1.001099308} + \frac{0.053158454}{1.001631476} + \frac{0.054950312}{1.002181876} + \right. \\ \left. \frac{0.054950312}{1.002732578} + \frac{0.054950312}{1.003283582} + \frac{0.053158454}{1.003816912} + \frac{100.054950312}{1.004368512} \right]$$

$$P = [0.054920133 + 0.054889971 + 0.053071869 + 0.054830678 + \\ 0.054800565 + 0.054770469 + 0.052956324 + 99.619760194]$$

$$P = 100.000000203 = \$100.000000$$

B. For newly issued floating rate notes      Formula:  
issued at a premium:

$$P = \sum_{i=1}^N \left( \frac{100 \times \frac{1}{360} (T_i - T_{i-1}) \times \max(r + s, 0)}{\prod_{k=1}^i \left( 1 + \frac{1}{360} (T_k - T_{k-1}) \times (r + m) \right)} \right) \\ + \frac{100}{\prod_{k=1}^N \left( 1 + \frac{1}{360} (T_k - T_{k-1}) \times (r + m) \right)}$$

Example:

The purpose of this example is to demonstrate how a floating rate note auction can result in a price at a premium given a

negative discount margin and spread at auction. For a new two-year floating rate note auctioned on July 25, 2012, and issued on July 31, 2012, with a maturity date of July 31, 2014,



solve for the price per 100 (P). In a new issue (not a reopening) of a floating rate note, the discount margin established at auction will be equal to the spread. In this example, the discount margin determined at auction is  $-0.150\%$ , but the floating rate note is subject to a daily interest rate accrual minimum of  $0.000\%$ .

Definitions:

$T_0$  = July 31, 2012.

$N = 8$ .

$T_N$  = July 31, 2014.

$r = 0.095022819\%$ .

$s = -0.150\%$ .

$m = -0.150\%$ .

As of the issue date the latest 13-week bill, auctioned at least two days prior, has the following information:

TABLE 1—13-WEEK BILL AUCTION DATA

Auction date	Issue date	Maturity date	Auction clearing price	Auction high rate	Index rate
7/23/2012	7/26/2012	10/25/2012	99.975986	0.095%	0.095022819%

### Computing the Index Rate

The index rate that equals the simple-interest money market yield on an actual/360 basis is computed as follows:

$$r = \frac{D}{1 - \frac{\Delta T}{360} D}$$

where  $D$  is the discount rate (or auction high rate), and  $\Delta T$  represents the number of days from (and including) the issue date of the 13-week bill to (but excluding) the maturity

date of the 13-week bill. In the table above,  $r = \frac{0.095\%}{1 - \frac{91}{360} \times 0.095\%} = 0.095022819\%$ .

### Computing the Projected Cash Flows

The following table presents the future interest payment dates and the number of days between them.

TABLE 2—PAYMENT DATES

Dates	Days between dates
Issue Date: $T_0 = 7/31/2012$ .	
1st Interest Date: $T_1 = 10/31/2012$ .....	$T_1 - T_0 = 92$
2nd Interest Date: $T_2 = 1/31/2013$ .....	$T_2 - T_1 = 92$
3rd Interest Date: $T_3 = 4/30/2013$ .....	$T_3 - T_2 = 89$
4th Interest Date: $T_4 = 7/31/2013$ .....	$T_4 - T_3 = 92$
5th Interest Date: $T_5 = 10/31/2013$ .....	$T_5 - T_4 = 92$
6th Interest Date: $T_6 = 1/31/2014$ .....	$T_6 - T_5 = 92$
7th Interest Date: $T_7 = 4/30/2014$ .....	$T_7 - T_6 = 89$
8th Interest & Maturity Dates: $T_8 = 7/31/2014$ .....	$T_8 - T_7 = 92$

Let

$$a_i = 100 \times \max(r + s, 0)/360$$

and

$$A_i = a_i \times (T_i - T_{i-1}) + 100 \times 1_{\{i=8\}}$$

$a_i$  Represents the daily projected interest, for a \$100 par value, that will accrue between the future interest payment dates  $T_{i-1}$  and  $T_i$  where  $i = 1, 2, \dots, 8$ .  $a_i$ 's are computed using the spread  $s = -0.150\%$ , and the fixed index

rate of  $r = 0.095022819\%$  applicable to the issue date (7/31/2012). For example:

$$a_i = 100 \times \max(0.00095022819 - 0.00150, 0)/360 = 100 \times 0/360 = 0.000000000$$

$A_i$  represents the projected cash flow the floating rate note holder will receive, for a \$100 par value, at the future interest payment date  $T_i$ , where  $i = 1, 2, \dots, 8$ .  $T_i - T_{i-1}$  is the number of days between the future interest payment dates  $T_{i-1}$  and  $T_i$ . To account

**Fiscal Service, Treasury**

**Pt. 356, App. B**

for the payback of the par value, the variable  $1_{(i=s)}$  takes the value 1 if the payment date is the maturity date, or 0 otherwise. For example:

$$A_1 = 92 \times 0.000000000 = 0.000000000$$

and

$$A_8 = 92 \times 0.000000000 + 100 = 100.000000000$$

Let

$$B_i = 1 + (r + m) \times (T_i - T_{i-1})/360$$

$B_i$  represents the projected compound factor between the future dates  $T_{i-1}$  and  $T_i$ , where  $i = 1, 2, \dots, 8$ . All  $B_i$ 's are computed

using the discount margin  $m = -0.150\%$  (equals the spread obtained at the auction), and the fixed index rate of  $r = 0.095022819\%$  applicable to the issue date (7/31/2012). For example:

$$B_3 = 1 + (0.00095022819 - 0.00150) \times 89/360 = 0.999864084.$$

The following table shows the projected daily accrued interests for \$100 par value ( $a_i$ 's), cash flows at interest payment dates ( $A_i$ 's), and the compound factors between payment dates ( $B_i$ 's).

**TABLE 3—PROJECTED CASH FLOWS AND COMPOUND FACTORS**

$i$	$a_i$	$A_i$	$B_i$
1 .....	0.000000000	0.000000000	0.999859503
2 .....	0.000000000	0.000000000	0.999859503
3 .....	0.000000000	0.000000000	0.999864084
4 .....	0.000000000	0.000000000	0.999859503
5 .....	0.000000000	0.000000000	0.999859503
6 .....	0.000000000	0.000000000	0.999859503
7 .....	0.000000000	0.000000000	0.999864084
8 .....	0.000000000	100.000000000	0.999859503

*Computing the Price*

The price is computed as follows:

$$\begin{aligned}
 P = & \left[ \frac{A_1}{B_1} + \frac{A_2}{B_1 B_2} + \frac{A_3}{B_1 B_2 B_3} + \frac{A_4}{B_1 B_2 B_3 B_4} + \frac{A_5}{B_1 B_2 B_3 B_4 B_5} + \right. \\
 & \left. \frac{A_6}{B_1 B_2 B_3 B_4 B_5 B_6} + \frac{A_7}{B_1 B_2 B_3 B_4 B_5 B_6 B_7} + \frac{A_8}{B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8} \right] \\
 P = & \left[ \frac{0.000000000}{B_1} + \frac{0.000000000}{B_1 B_2} + \frac{0.000000000}{B_1 B_2 B_3} + \frac{0.000000000}{B_1 B_2 B_3 B_4} + \right. \\
 & \left. \frac{0.000000000}{B_1 B_2 B_3 B_4 B_5} + \frac{0.000000000}{B_1 B_2 B_3 B_4 B_5 B_6} + \frac{0.000000000}{B_1 B_2 B_3 B_4 B_5 B_6 B_7} + \frac{100.000000000}{B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8} \right] \\
 P = & [0.000000000 + 0.000000000 + 0.000000000 + 0.000000000 + \\
 & 0.000000000 + 0.000000000 + 0.000000000 + 100.000000000/0.998885730] \\
 P = & 100.111551298 = \$100.111551
 \end{aligned}$$

*Definitions for Reopenings of Floating Rate Notes and Calculation of Interest Payments*

$IP_i$  = the quarterly interest payment at date  $T_i$ .  
 $P_D$  = the price that includes the accrued interest per \$100 par value as of the reopening issue date.  
 $AI$  = accrued interest per \$100 par value as of the reopening issue date.  
 $P_C$  = the price without accrued interest per \$100 par value as of the reopening issue date.  
 $T_{-1}$  = the dated date if the reopening occurs before the first interest payment date, or, otherwise, the latest interest payment date prior to the reopening issue date.  
 $T_0$  = the reopening issue date.  
 $N$  = the total number of remaining quarterly interest payments as of the reopening issue date.  
 $i$  and  $k$  = indexes that identify the sequence of interest payment dates relative to the issue date. For example  $T_1$ ,  $T_2$ , and  $T_3$

represent the first, second, and the third interest payment dates after the issue date respectively, while  $T_{-1}$  represents the preceding interest payment date before the issue date.

$j$  = an index that identifies days between consecutive interest payment dates.

$T_i$  = the  $i^{\text{th}}$  remaining quarterly interest payment date.

$T_i - T_{i-1}$  = the number of days between the interest payment date  $T_i$  and the preceding interest payment date.

$T_N$  = the maturity date.

$r_j$ 's = the effective index rates for days between the last interest payment date and the reopening issue date.

$r$  = the index rate applicable to the reopening issue date.

$s$  = the spread.

$m$  = the discount margin.

C. Pricing and accrued interest for reopened floating rate notes

Formula:

$$P_D = \frac{100 \times \frac{1}{360} \sum_{j=T_{-1}}^{T_0-1} \max(r_j + s, 0)}{1 + \frac{1}{360}(T_1 - T_0) \times (r + m)} + \sum_{i=1}^N \left( \frac{100 \times \frac{1}{360}(T_i - T_{i-1}) \times \max(r + s, 0)}{\prod_{k=1}^i \left(1 + \frac{1}{360}(T_k - T_{k-1}) \times (r + m)\right)} \right) + \frac{100}{\prod_{k=1}^N \left(1 + \frac{1}{360}(T_k - T_{k-1}) \times (r + m)\right)} AI = 100 \times \frac{1}{360} \sum_{j=T_{-1}}^{T_0-1} \max(r_j + s, 0)$$

**Example:**

The purpose of this example is to determine the floating rate note prices with and without accrued interest at the time of the reopening auction. For a two-year floating rate note that was originally auctioned on July 25, 2012, with an issue date of July 31, 2012, reopened in an auction on August 30, 2012 and issued on August 31, 2012, with a ma-

turity date of July 31, 2014, solve for accrued interest per 100 ( $AI$ ), the price with accrued interest per 100 ( $P_D$ ) and the price without accrued interest per 100 ( $P_C$ ). Since this is a reopening of an original issue from the prior month, Table 2 as shown in the example is used for accrued interest calculations. In the case of floating rate note reopenings, the spread on the security remains equal to the

**Fiscal Service, Treasury**

**Pt. 356, App. B**

spread that was established at the original auction of the floating rate notes.

Definitions:

$T_{-1}$  = July 31, 2012.

$T_0$  = August 31, 2012.

$N = 8$ .

$T_N$  = July 31, 2014.

$r = 0.105027876\%$ .

$s = 0.120\%$ .

$m = 0.100\%$ .

The following table shows the past results for the 13-week bill auction.

**TABLE 1—13-WEEK BILL AUCTION DATA**

Auction date	Issue date	Maturity date	Auction clearing price	Auction high rate (percent)	Index rate (percent)
7/23/2012 .....	7/26/2012	10/25/2012	99.975986	0.095	0.095022819
7/30/2012 .....	8/2/2012	11/1/2012	99.972194	0.110	0.110030595
8/6/2012 .....	8/9/2012	11/8/2012	99.974722	0.100	0.100025284
8/13/2012 .....	8/16/2012	11/15/2012	99.972194	0.110	0.110030595
8/20/2012 .....	8/23/2012	11/23/2012	99.973167	0.105	0.105028183
8/27/2012 .....	8/30/2012	11/29/2012	99.973458	0.105	0.105027876

*Computing the Index Rate*

The index rate that equals the simple-interest money market yield on an actual/360 basis is computed as follows:

$$r = \frac{D}{1 - \frac{\Delta T}{360} D}$$

where  $D$  is the discount rate (or auction high rate), and  $\Delta T$  represents the number of days from (and including) the issue date of the 13-week bill to (but excluding) the maturity date of the 13-week bill. In the table above the corresponding index rate for the

$$8/27/2012 \text{ auction is } r = \frac{0.105\%}{1 - \frac{91}{360} \times 0.105\%} = 0.105027876\%$$

The following table shows the index rates applicable for the accrued interest.

**TABLE 2—APPLICABLE INDEX RATE**

Accrual starts	Accrual ends	Number of days in accrual period	Applicable floating rate	
			Auction date	Index rate (percent)
7/31/2012 .....	7/31/2012	1	7/23/2012	0.095022819
8/1/2012 .....	8/6/2012	6	7/30/2012	0.110030595
8/7/2012 .....	8/13/2012	7	8/6/2012	0.100025284
8/14/2012 .....	8/20/2012	7	8/13/2012	0.110030595
8/21/2012 .....	8/27/2012	7	8/20/2012	0.105028183
8/28/2012 .....	8/30/2012	3	8/27/2012	0.105027876

*Computing the Accrued Interest*

The accrued interest as of the new issue date (8/31/2012) for a \$100 par value is:

$$\begin{aligned} AI &= 1 \times 100 \times \max(0.00095022819 + 0.00120, 0)/360 \\ &+ 6 \times 100 \times \max(0.00110030595 + 0.00120, 0)/360 \\ &+ 7 \times 100 \times \max(0.00100025284 + 0.00120, 0)/360 \\ &+ 7 \times 100 \times \max(0.00110030595 + 0.00120, 0)/360 \\ &+ 7 \times 100 \times \max(0.00105028183 + 0.00120, 0)/360 \\ &+ 3 \times 100 \times \max(0.00105027876 + 0.00120, 0)/360 \end{aligned}$$

$$AI = 1 \times 0.000597286$$

$$+ 6 \times 0.000638974$$

$$+ 7 \times 0.000611181$$

$$+ 7 \times 0.000638974$$

$$+ 7 \times 0.000625078$$

$$+ 3 \times 0.000625077$$

$$AI = 0.000597286 + 0.003833844 + 0.004278267 + 0.004472818 + 0.004375546 + 0.001875231$$

$$AI = 0.019432992 = \$0.019433$$

*Computing the Projected Cash Flows*

The following table presents the future interest payment dates and the number of days between them.

TABLE 3—PAYMENT DATES

Dates	Days between dates
Original Issue Date: $T_{-1} = 7/31/2012$ .	
New Issue Date: $T_0 = 8/31/2012$ .....	$T_0 - T_{-1} = 31$
1st Interest Date: $T_1 = 10/31/2012$ .....	$T_1 - T_0 = 61$
2nd Interest Date: $T_2 = 1/31/2013$ .....	$T_2 - T_1 = 92$
3rd Interest Date: $T_3 = 4/30/2013$ .....	$T_3 - T_2 = 89$
4th Interest Date: $T_4 = 7/31/2013$ .....	$T_4 - T_3 = 92$
5th Interest Date: $T_5 = 10/31/2013$ .....	$T_5 - T_4 = 92$
6th Interest Date: $T_6 = 1/31/2014$ .....	$T_6 - T_5 = 92$
7th Interest Date: $T_7 = 4/30/2014$ .....	$T_7 - T_6 = 89$
8th Interest & Maturity Dates: $T_8 = 7/31/2014$ .....	$T_8 - T_7 = 92$

Let

$$a_i = 100 \times \max(r + s, 0)/360$$

and

$$A_i = a_i \times (T_i - T_{i-1}) + 100 \times 1_{\{i=8\}}$$

$a_i$  represents the daily projected interest, for a \$100 par value, that will accrue between the future interest payment dates  $T_{i-1}$  and  $T_i$ , where  $i=1,2,\dots,8$ .  $a_i$ 's are computed using the spread  $s = 0.120\%$  obtained at the original auction, and the fixed index rate of  $r = 0.105027876\%$  applicable to the new issue date (8/31/2012). For example:

$$a_i = 100 \times \max(0.00105027876 + 0.00120, 0)/360 = 0.000625077$$

$A_i$  represents the projected cash flow the floating rate note holder will receive, less accrued interest, for a \$100 par value, at the future interest payment date  $T_i$ , where  $i=1,2,\dots,8$ .  $T_i - T_{i-1}$  is the number of days between the future interest payment dates  $T_{i-1}$  and  $T_i$ . To account for the payback of the par value, the variable  $1_{\{i=8\}}$  takes the value 1 if

the payment date is the maturity date, or 0 otherwise. For example:

$$A_i = 61 \times 0.000625077 = 0.038129697$$

and

$$A_8 = 92 \times 0.000625077 + 100 = 100.057507084$$

Let

$$B_i = 1 + (r + m) \times (T_i - T_{i-1})/360$$

$B_i$  represents the projected compound factor between the future dates  $T_{i-1}$  and  $T_i$ , where  $i=1,2,\dots,8$ . All  $B_i$ 's are computed using the discount margin  $m = 0.100\%$  obtained at the reopening auction, and the fixed index rate of  $r = 0.105027876\%$  applicable to the new issue date (8/31/2012). For example:

$$B_3 = 1 + (0.00105027876 + 0.00100) \times 89/360 = 1.000506874$$

The following table shows the projected daily accrued interests for \$100 par value ( $a_i$ 's), cash flows at interest payment dates ( $A_i$ 's), and the compound factors between payment dates ( $B_i$ 's).

TABLE 4—PROJECTED CASH FLOWS AND COMPOUND FACTORS

$i$	$a_i$	$A_i$	$B_i$
1 .....	0.000625077	0.038129697	1.000347408
2 .....	0.000625077	0.057507084	1.000523960
3 .....	0.000625077	0.055631853	1.000506874
4 .....	0.000625077	0.057507084	1.000523960
5 .....	0.000625077	0.057507084	1.000523960
6 .....	0.000625077	0.057507084	1.000523960
7 .....	0.000625077	0.055631853	1.000506874
8 .....	0.000625077	100.057507084	1.000523960

*Computing the Price*

The price with accrued interest is computed as follows:

$$\begin{aligned}
P_D &= \left[ \frac{AI + A_1}{B_1} + \frac{A_2}{B_1 B_2} + \frac{A_3}{B_1 B_2 B_3} + \frac{A_4}{B_1 B_2 B_3 B_4} + \frac{A_5}{B_1 B_2 B_3 B_4 B_5} + \right. \\
&\quad \left. \frac{A_6}{B_1 B_2 B_3 B_4 B_5 B_6} + \frac{A_7}{B_1 B_2 B_3 B_4 B_5 B_6 B_7} + \frac{A_8}{B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8} \right] \\
P_D &= \left[ \frac{0.019432992 + 0.038129697}{B_1} + \frac{0.057507084}{B_1 B_2} + \frac{0.055631853}{B_1 B_2 B_3} + \frac{0.057507084}{B_1 B_2 B_3 B_4} + \right. \\
&\quad \left. \frac{0.057507084}{B_1 B_2 B_3 B_4 B_5} + \frac{0.057507084}{B_1 B_2 B_3 B_4 B_5 B_6} + \frac{0.055631853}{B_1 B_2 B_3 B_4 B_5 B_6 B_7} + \frac{100.057507084}{B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8} \right] \\
P_D &= \left[ \frac{0.057562689}{1.000347408} + \frac{0.057507084}{1.000871550} + \frac{0.055631853}{1.001378866} + \frac{0.057507084}{1.001903548} + \right. \\
&\quad \left. \frac{0.057507084}{1.002428506} + \frac{0.057507084}{1.002953738} + \frac{0.055631853}{1.003462109} + \frac{100.057507084}{1.003987883} \right] \\
P_D &= [0.057542698 + 0.057457007 + 0.055555250 + 0.057397824 + \\
&\quad 0.057367766 + 0.057337723 + 0.055439914 + 99.660074368] \\
P_D &= 100.058172550 = \$100.058173
\end{aligned}$$

The price without accrued interest is computed as follows:

$$P_C = P_D - AI = 100.058172550 - 0.019432992$$

$$P_C = 100.038739558 = \$100.038740$$

D. For calculating interest payments:  
Example:

For calculating interest payments:  
Example:

For a new issue of a two-year floating rate note auctioned on July 25, 2012, and issued on July 31, 2012, with a maturity date of July 31, 2014, and a first interest payment date of October 31, 2012, calculate the quarterly inter-

est payments (IP<sub>i</sub>) per 100. In a new issuance (not a reopening) of a new floating rate note, the discount margin determined at auction will be equal to the spread. The interest accrual rate used for this floating rate note on the issue date is 0.215022819% (index rate of 0.095022819% plus a spread of 0.120%) and this rate is used for each daily interest accrual over the life of the security for the purposes of this example.

- (a) If it is a new floating rate note, then  $IP_i = 100 \times \frac{1}{360} (T_i - T_{i-1}) \times \max(r + s, 0)$
- (b) If it is a reopened floating rate note, and the interest payment is the first one after the reopening, then  $IP_i = 100 \times \frac{1}{360} \sum_{j=T_{i-1}}^{T_0} \max(r_j + s, 0) + 100 \times \frac{1}{360} (T_i - T_0) \times \max(r + s, 0)$
- (c) If it is a reopened floating rate note, and the interest payment is not the first one after the reopening, then  $IP_i = 100 \times \frac{1}{360} (T_i - T_{i-1}) \times \max(r + s, 0)$

*Example 1:* Projected interest payment as of the original issue date.  $s = 0.120\%$ .  
 $T_0 =$  July 31, 2012.  $m = 0.120\%$ .  
 $N = 8$ .  
 $T_N =$  July 31, 2014.  
 $r = 0.095022819\%$ .

As of the issue date the latest 13-week bill, auctioned at least two days prior, has the following information:

TABLE 1—13-WEEK BILL AUCTION DATA

Auction date	Issue date	Maturity date	Auction clearing price	Auction high rate	Index rate
7/23/2012 .....	7/26/2012	10/25/2012	99.975986	0.095%	0.095022819%

#### Computing the Index Rate

The index rate that equals the simple-interest money market yield on an actual/360 basis is computed as follows:

$$r = \frac{D}{1 - \frac{\Delta T}{360} D}$$

where  $D$  is the discount rate (or auction high rate), and  $\Delta T$  represents the number of days from (and including) the issue date of the 13-week bill to (but excluding) the maturity date of the 13-week bill. In the table above,  $r = \frac{0.095\%}{1 - \frac{91}{360} \times 0.095\%} = 0.095022819\%$ .

#### Computing the Projected Cash Flows

The following table presents the future interest payment dates and the number of days between them.

TABLE 2—PAYMENT DATES

Dates	Days between dates
Issue Date: $T_0 = 7/31/2012$ .	
1st Interest Date: $T_1 = 10/31/2012$ .....	$T_1 - T_0 = 92$
2nd Interest Date: $T_2 = 1/31/2013$ .....	$T_2 - T_1 = 92$
3rd Interest Date: $T_3 = 4/30/2013$ .....	$T_3 - T_2 = 89$
4th Interest Date: $T_4 = 7/31/2013$ .....	$T_4 - T_3 = 92$
5th Interest Date: $T_5 = 10/31/2013$ .....	$T_5 - T_4 = 92$
6th Interest Date: $T_6 = 1/31/2014$ .....	$T_6 - T_5 = 92$
7th Interest Date: $T_7 = 4/30/2014$ .....	$T_7 - T_6 = 89$
8th Interest & Maturity Dates: $T_8 = 7/31/2014$ .....	$T_8 - T_7 = 92$

Using the spread  $s = 0.120\%$ , and the fixed index rate of  $r = 0.095022819\%$  applicable to the issue date (7/31/2012), the first and seventh projected interest payments are computed as follows:

$$IP_1 = 92 \times [100 \times \max(0.00095022819 + 0.00120, 0) / 360]$$

$$IP_1 = 92 \times 0.000597286 = 0.054950312$$

$$IP_7 = 89 \times [100 \times \max(0.00095022819 + 0.00120, 0) / 360]$$

$$IP_7 = 89 \times 0.000597286 = 0.053158454$$

The following table shows all projected interest payments as of the issue date.

TABLE 3—PROJECTED INTEREST PAYMENTS

$i$	Dates	$IP_i$
1 .....	10/31/2012	0.054950312
2 .....	1/31/2013	0.054950312
3 .....	4/30/2013	0.053158454
4 .....	7/31/2013	0.054950312
5 .....	10/31/2013	0.054950312
6 .....	1/31/2014	0.054950312
7 .....	4/30/2014	0.053158454
8 .....	7/31/2014	0.054950312

*Example 2:* Projected interest payment as of the reopening issue date (intermediate values, including rates in percentage terms, are rounded to nine decimal places).

This example demonstrates the calculations required to determine the interest payment due when the reopened floating rate note is issued. This example also demonstrates the need to calculate accrued interest at the time of a floating rate reopening auction. Since this is a reopening of an original issue from 31 days prior, Table 5 as shown in the example is used for accrued interest calculations. For a two-year floating rate note originally auctioned on July 25, 2012 with an original issue date of July 31, 2012, reopened by an auction on August 30, 2012 and issued on August 31, 2012, with a maturity date of July 31, 2014, calculate the quarterly interest payments ( $IP_i$ ) per 100.  $T_{-1}$  is the dated date if the reopening occurs before the first interest payment date, or otherwise the latest interest payment date prior to the new issue date.

$T_{-1}$  = July 31, 2012.

$T_0$  = August 31, 2012.

$N = 8$ .

$T_N$  = July 31, 2014.

$r = 0.105027876\%$ .

$s = 0.120\%$ .

$m = 0.100\%$ .

The following table shows the past results for the 13-week bill auction.

TABLE 4—13-WEEK BILL AUCTION DATA

Auction date	Issue date	Maturity date	Auction clearing price	Auction high rate (percent)	Index rate (percent)
7/23/2012 .....	7/26/2012	10/25/2012	99.975986	0.095	0.095022819
7/30/2012 .....	8/2/2012	11/1/2012	99.972194	0.110	0.110030595
8/6/2012 .....	8/9/2012	11/8/2012	99.974722	0.100	0.100025284
8/13/2012 .....	8/16/2012	11/15/2012	99.972194	0.110	0.110030595
8/20/2012 .....	8/23/2012	11/23/2012	99.973167	0.105	0.105028183
8/27/2012 .....	8/30/2012	11/29/2012	99.973458	0.105	0.105027876

### Computing the Index Rate

The index rate that equals the simple-interest money market yield on an actual/360 basis is computed as follows:

$$r = \frac{D}{1 - \frac{\Delta T}{360} D}$$

where  $D$  is the discount rate (or auction high rate), and  $\Delta T$  represents the number of days from (and including) the issue date of the 13-week bill to (but excluding) the maturity date of the 13-week bill. In the table above the corresponding index rate for the

$$7/23/2012 \text{ auction is } r = \frac{0.095\%}{1 - \frac{91}{360} \times 0.095\%} = 0.095022819\%.$$

The following table shows the index rates applicable for the accrued interest.



TABLE 5—APPLICABLE INDEX RATE

Accrual starts	Accrual ends	Number of days in accrual period	Applicable floating rate	
			Auction date	Index rate (percent)
7/31/2012 .....	7/31/2012	1	7/23/2012	0.095022819
8/1/2012 .....	8/6/2012	6	7/30/2012	0.110030595
8/7/2012 .....	8/13/2012	7	8/6/2012	0.100025284
8/14/2012 .....	8/20/2012	7	8/13/2012	0.110030595
8/21/2012 .....	8/27/2012	7	8/20/2012	0.105028183
8/28/2012 .....	8/30/2012	3	8/27/2012	0.105027876

*Computing the accrued interest*

The accrued interest as of 8/31/2012 for a \$100 par value is:

$$\begin{aligned}
 AI &= 1 \times 100 \times \max(0.00095022819 + 0.00120, 0)/360 \\
 &+ 6 \times 100 \times \max(0.00110030595 + 0.00120, 0)/360 \\
 &+ 7 \times 100 \times \max(0.00100025284 + 0.00120, 0)/360 \\
 &+ 7 \times 100 \times \max(0.00110030595 + 0.00120, 0)/360 \\
 &+ 7 \times 100 \times \max(0.00105028183 + 0.00120, 0)/360 \\
 &+ 3 \times 100 \times \max(0.00105027876 + 0.00120, 0)/360 \\
 AI &= 1 \times 0.000597286 \\
 &+ 6 \times 0.000638974
 \end{aligned}$$

$$+ 7 \times 0.000611181$$

$$+ 7 \times 0.000638974$$

$$+ 7 \times 0.000625078$$

$$+ 3 \times 0.000625077$$

$$AI = 0.000597286 + 0.003833844 + 0.004278267 + 0.004472818 + 0.004375546 + 0.001875231$$

$$AI = 0.019432992 = \$0.019433$$

The following table presents the future interest payment dates and the number of days between them.

TABLE 6—PAYMENT DATES

Dates	Days between dates
Original Issue Date: $T_{-1} = 7/31/2012$ .....	
New Issue Date: $T_0 = 8/31/2012$ .....	$T_0 - T_{-1} = 31$
1st Interest Date: $T_1 = 10/31/2012$ .....	$T_1 - T_0 = 61$
2nd Interest Date: $T_2 = 1/31/2013$ .....	$T_2 - T_1 = 92$
3rd Interest Date: $T_3 = 4/30/2013$ .....	$T_3 - T_2 = 89$
4th Interest Date: $T_4 = 7/31/2013$ .....	$T_4 - T_3 = 92$
5th Interest Date: $T_5 = 10/31/2013$ .....	$T_5 - T_4 = 92$
6th Interest Date: $T_6 = 1/31/2014$ .....	$T_6 - T_5 = 92$
7th Interest Date: $T_7 = 4/30/2014$ .....	$T_7 - T_6 = 89$
8th Interest & Maturity Dates: $T_8 = 7/31/2014$ .....	$T_8 - T_7 = 92$

Using the original spread  $s = 0.120\%$  (obtained on 7/25/2012), and the fixed index rate of  $r = 0.105027876\%$  applicable to the new issue date (8/31/2012), the first and eighth projected interest payments are computed as follows:

$$IP_1 = 0.019432992 + 61 \times [100 \times \max(0.00105027876 + 0.00120, 0)/360]$$

$$IP_1 = 0.019432992 + 61 \times 0.000625077$$

$$IP_1 = 0.019432992 + 0.038129697 = 0.057562689$$

and

$$IP_8 = 92 \times [100 \times \max(0.00105027876 + 0.00120, 0)/360]$$

$$IP_8 = 92 \times 0.000625077 = 0.057507084$$

The following table shows all projected interest payments as of the new issue date.

TABLE 7—PROJECTED INTEREST PAYMENTS

$i$	Dates	$IP_i$
1 .....	10/31/2012	0.057562689
2 .....	1/31/2013	0.057507084
3 .....	4/30/2013	0.055631853
4 .....	7/31/2013	0.057507084
5 .....	10/31/2013	0.057507084

TABLE 7—PROJECTED INTEREST PAYMENTS—Continued

$i$	Dates	$IP_i$
6 .....	1/31/2014	0.057507084
7 .....	4/30/2014	0.055631853
8 .....	7/31/2014	0.057507084

*Definitions for Newly Issued Floating Rate Notes with an Issue Date that Occurs after the Dated Date*

$P_D$  = the price that includes accrued interest from the dated date to the issue date per \$100 par value as of the issue date.

$AI$  = the accrued interest per \$100 par value as of the issue date.

$P_C$  = the price without accrued interest per \$100 par value as of the issue date.

$T_{-1}$  = the dated date.

$T_0$  = the issue date.

$N$  = the total number of remaining quarterly interest payments as of the new issue date.

**Fiscal Service, Treasury**

**Pt. 356, App. B**

$i$  and  $k$  = indexes that identify the sequence of interest payment dates.

$j$  = an index that identifies days between the dated date and the issue date.

$T_i$  = the  $i^{\text{th}}$  quarterly future interest payment date.

$T_i - T_{i-1}$  = the number of days between the interest payment date  $T_i$  and the preceding interest payment date.

$T_N$  = the maturity date.

$r_j$ 's = the effective index rates for days between the dated date and the issue date.

$r$  = the index rate applicable to the issue date.

$s$  = the spread.

$m$  = the discount margin.

E. Pricing and accrued interest for new issue floating rate notes with an issue date that occurs after the dated date

Formula:

$$P_D = \frac{100 \times \frac{1}{360} \sum_{j=T_0-1}^{T_0-1} \max(r_j + s, 0)}{1 + \frac{1}{360} (T_1 - T_0) \times (r + m)} + \sum_{i=1}^N \left( \frac{100 \times \frac{1}{360} (T_i - T_{i-1}) \times \max(r + s, 0)}{\prod_{k=1}^i \left( 1 + \frac{1}{360} (T_k - T_{k-1}) \times (r + m) \right)} \right) + \frac{100}{\prod_{k=1}^N \left( 1 + \frac{1}{360} (T_k - T_{k-1}) \times (r + m) \right)}$$

$$AI = 100 \times \frac{1}{360} \sum_{j=T_0-1}^{T_0-1} \max(r_j + s, 0)$$

**Example:**

The purpose of this example is to demonstrate how a floating rate note can have a price without accrued interest of less than \$100 par value when the issue date occurs after the dated date. An original issue two-year floating rate note is auctioned on December 29, 2011, with a dated date of December 31, 2011, an issue date of January 3, 2012, and a maturity date of December 31, 2013.

**Definitions:**

Dated date = 12/31/2011.

Issue date = 1/3/2012.

Maturity date = 12/31/2013.

Spread = 1.000% at auction.

Discount margin = 1.000%.

As of the issue date the latest 13-week bill, auctioned at least two days prior, has the following information:

**TABLE 1—13-WEEK BILL AUCTION DATA**

Auction date	Issue date	Maturity date	Auction clearing price	Auction high rate	Index rate
12/27/2011 .....	12/29/2011	3/29/2012	99.993681	0.025%	0.025001580%

*Computing the Index Rate*

The index rate that equals the simple-interest money market yield on an actual/360 basis is computed as follows:

$$r = \frac{D}{1 - \frac{\Delta T}{360} D}$$

where  $D$  is the discount rate (or auction high rate), and  $\Delta T$  represents the number of days from (and including) the issue date of the 13-week bill to (but excluding) the maturity date of the 13-week bill. In the table above the corresponding index rate for the

$$12/27/2011 \text{ auction is } r = \frac{0.025\%}{1 - \frac{91}{360} \times 0.025\%} = 0.025001580\%$$

The following table shows the index rates applicable for the accrued interest.

TABLE 2—APPLICABLE INDEX RATE

Accrual starts	Accrual ends	Number of days in accrual period	Applicable floating rate	
			Auction date	Index rate
12/31/2011 .....	1/2/2012	3	12/27/2011	0.025001580%

*Computing the accrued interest*

$$AI = 0.008541681 = \$0.008542$$

The accrued interest as of the new issue date (1/3/2012) for a \$100 par value is:

$$AI = 3 \times 100 \times \max(0.00025001580 + 0.01000, 0)/360$$

$$AI = 3 \times 0.002847227$$

*Computing the Projected Cash Flows*

The following table presents the future interest payment dates and the number of days between them.

TABLE 3—PAYMENT DATES

Dates	Days between dates
Dated Date: $T_{-1} = 12/31/2011$ .....	
Issue Date: $T_0 = 1/3/2012$ .....	$T_0 - T_{-1} = 3$
1st Interest Date: $T_1 = 3/31/2012$ .....	$T_1 - T_0 = 88$
2nd Interest Date: $T_2 = 6/30/2012$ .....	$T_2 - T_1 = 91$
3rd Interest Date: $T_3 = 9/30/2012$ .....	$T_3 - T_2 = 92$
4th Interest Date: $T_4 = 12/31/2012$ .....	$T_4 - T_3 = 92$
5th Interest Date: $T_5 = 3/31/2013$ .....	$T_5 - T_4 = 90$
6th Interest Date: $T_6 = 6/30/2013$ .....	$T_6 - T_5 = 91$
7th Interest Date: $T_7 = 9/30/2013$ .....	$T_7 - T_6 = 92$
8th Interest & Maturity Dates: $T_8 = 12/31/2013$ .....	$T_8 - T_7 = 92$

Let

$$a_i = 100 \times \max(r + s, 0)/360$$

and

$$A_i = a_i \times (T_i - T_{i-1}) + 100 \times 1_{\{i=8\}}$$

$a_i$  represents the daily projected interest, for a \$100 par value, that will accrue between the future interest payment dates  $T_{i-1}$  and  $T_i$ , where  $i = 1, 2, \dots, 8$ .  $a_i$ 's are computed using the spread  $s = 1.000\%$  obtained at the auction, and the fixed index rate of  $r = 0.025001580\%$

applicable to the issue date (1/3/2012). For example:

$$a_1 = 100 \times \max(0.00025001580 + 0.01000, 0)/360 = 0.002847227$$

$A_i$  represents the projected cash flow the floating rate note holder will receive, less accrued interest, for a \$100 par value, at the future interest payment date  $T_i$ , where  $i = 1, 2, \dots, 8$ .  $T_i - T_{i-1}$  is the number of days between the future interest payment dates  $T_{i-1}$

and  $T_i$ . To account for the payback of the par value, the variable  $1_{(i=8)}$  takes the value 1 if the payment date is the maturity date, or 0 otherwise. For example:

$$A_1 = 88 \times 0.002847227 = 0.250555976$$

and

$$A_8 = 92 \times 0.002847227 + 100 = 100.261944884$$

Let

$$B_i = 1 + (r + m) \times (T_i - T_{i-1})/360$$

$B_i$  represents the projected compound factor between the future dates  $T_{i-1}$  and  $T_i$ ,

where  $i = 1, 2, \dots, 8$ . All  $B_i$ 's are computed using the discount margin  $m = 1.000\%$  (equals the spread obtained at the auction), and the fixed index rate of  $r = 0.025001580\%$  applicable to the issue date (1/3/2012). For example:

$$B_3 = 1 + (0.00025001580 + 0.01000) \times 92/360 = 1.002619448$$

The following table shows the projected daily accrued interests for \$100 par value ( $a_i$ 's), cash flows at interest payment dates ( $A_i$ 's), and the compound factors between payment dates ( $B_i$ 's).

TABLE 4—PROJECTED CASH FLOWS AND COMPOUND FACTORS

$i$	$a_i$	$A_i$	$B_i$
1 .....	0.002847227	0.250555976	1.002505559
2 .....	0.002847227	0.259097657	1.002590976
3 .....	0.002847227	0.261944884	1.002619448
4 .....	0.002847227	0.261944884	1.002619448
5 .....	0.002847227	0.256250430	1.002562504
6 .....	0.002847227	0.259097657	1.002590976
7 .....	0.002847227	0.261944884	1.002619448
8 .....	0.002847227	100.261944884	1.002619448

#### *Computing the price*

The price with accrued interest is computed as follows:

$$\begin{aligned}
P_D &= \left[ \frac{AI + A_1}{B_1} + \frac{A_2}{B_1 B_2} + \frac{A_3}{B_1 B_2 B_3} + \frac{A_4}{B_1 B_2 B_3 B_4} + \frac{A_5}{B_1 B_2 B_3 B_4 B_5} + \right. \\
&\quad \left. \frac{A_6}{B_1 B_2 B_3 B_4 B_5 B_6} + \frac{A_7}{B_1 B_2 B_3 B_4 B_5 B_6 B_7} + \frac{A_8}{B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8} \right] \\
P_D &= \left[ \frac{0.008541681 + 0.250555976}{B_1} + \frac{0.259097657}{B_1 B_2} + \frac{0.261944884}{B_1 B_2 B_3} + \frac{0.261944884}{B_1 B_2 B_3 B_4} + \right. \\
&\quad \left. \frac{0.256250430}{B_1 B_2 B_3 B_4 B_5} + \frac{0.259097657}{B_1 B_2 B_3 B_4 B_5 B_6} + \frac{0.261944884}{B_1 B_2 B_3 B_4 B_5 B_6 B_7} + \frac{100.261944884}{B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8} \right] \\
P_D &= \left[ \frac{0.259097657}{1.002505559} + \frac{0.259097657}{1.005103027} + \frac{0.261944884}{1.007735842} + \frac{0.261944884}{1.010375554} + \right. \\
&\quad \left. \frac{0.256250430}{1.012964645} + \frac{0.259097657}{1.015589212} + \frac{0.261944884}{1.018249495} + \frac{100.261944884}{1.020916747} \right] \\
P_D &= [0.258450095 + 0.257782188 + 0.259934075 + 0.259254970 + \\
&\quad 0.252970754 + 0.255120529 + 0.257250198 + 98.207758055] \\
P_D &= 100.008520864 = \$100.008521
\end{aligned}$$

The price without accrued interest is computed as follows:

$$P_C = P_D - AI = 100.008520864 - 0.008541681$$

$$P_C = 99.999979183 = \$99.999979$$

#### V. COMPUTATION OF ADJUSTED VALUES AND PAYMENT AMOUNTS FOR STRIPPED INFLATION-PROTECTED INTEREST COMPONENTS

NOTE: Valuing an interest component stripped from an inflation-protected security at its adjusted value enables this interest component to be interchangeable (fungible) with other interest components that have the same maturity date, regardless of the underlying inflation-protected security from which the interest components were stripped. The adjusted value provides for fungibility of these various interest components when buying, selling, or transferring them or when reconstituting an inflation-protected security.

#### Definitions:

c = C/100 = the regular annual interest rate, payable semiannually, e.g., .03625 (the decimal equivalent of a 3%% interest rate)

Par = par amount of the security to be stripped

Ref CPI<sub>IssueDate</sub> = reference CPI for the original issue date (or dated date, when the dated date is different from the original issue date) of the underlying (unstripped) security

Ref CPI<sub>Date</sub> = reference CPI for the maturity date of the interest component

AV = adjusted value of the interest component

PA = payment amount at maturity by Treasury

## Fiscal Service, Treasury

## Pt. 356, App. B

### Formulas:

AV = Par(C/2)(100/Ref CPI<sub>IssueDate</sub>) (rounded to 2 decimals with no intermediate rounding)  
 PA = AV(Ref CPI<sub>Date</sub>/100) (rounded to 2 decimals with no intermediate rounding)

### Example:

A 10-year inflation-protected note paying 3½% interest was issued on January 15, 1999, with the second interest payment on January 15, 2000. The Ref CPI of January 15, 1999 (Ref CPI<sub>IssueDate</sub>) was 164.00000, and the Ref CPI on January 15, 2000 (Ref CPI<sub>Date</sub>) was 168.24516. Calculate the adjusted value and the payment amount at maturity of the interest component.

### Definitions:

c = .03875  
 Par = \$1,000,000  
 Ref CPI<sub>IssueDate</sub> = 164.00000  
 Ref CPI<sub>Date</sub> = 168.24516

### Resolution:

For a par amount of \$1 million, the adjusted value of each stripped interest component was \$1,000,000(.03875/2)(100/164.00000), or \$11,814.02 (no intermediate rounding).

For an interest component that matured on January 15, 2000, the payment amount was \$11,814.02 (168.24516/100), or \$19,876.52 (no intermediate rounding).

## VI. COMPUTATION OF PURCHASE PRICE, DISCOUNT RATE, AND INVESTMENT RATE (COUPON-EQUIVALENT YIELD) FOR TREASURY BILLS

### A. Conversion of the discount rate to a purchase price for Treasury bills of all maturities:

#### Formula:

$P = 100 (1 - dr / 360)$ .

#### Where:

d = discount rate, in decimals.  
 r = number of days remaining to maturity.  
 P = price per 100 (dollars).

#### Example:

For a bill issued November 24, 1989, due February 22, 1990, at a discount rate of 7.610%, solve for price per 100 (P).

#### Definitions:

d = .07610.  
 r = 90 (November 24, 1989 to February 22, 1990).

#### Resolution:

$P = 100 (1 - dr / 360)$ .  
 (1)  $P = 100 [1 - (.07610)(90) / 360]$ .  
 (2)  $P = 100 (1 - .019025)$ .  
 (3)  $P = 100 (.980975)$ .  
 (4)  $P = 98.097500$ .

NOTE: Purchase prices per \$100 are rounded to six decimal places, using normal rounding procedures.

### B. Computation of purchase prices and discount amounts based on price per \$100, for Treasury bills of all maturities:

1. To determine the purchase price of any bill, divide the par amount by 100 and multiply the resulting quotient by the price per \$100.

#### Example:

To compute the purchase price of a \$10,000 13-week bill sold at a price of \$98.098000 per \$100, divide the par amount (\$10,000) by 100 to obtain the multiple (100). That multiple times 98.098000 results in a purchase price of \$9,809.80.

2. To determine the discount amount for any bill, subtract the purchase price from the par amount of the bill.

#### Example:

For a \$10,000 bill with a purchase price of \$9,809.80, the discount amount would be \$190.20, or \$10,000 - \$9,809.80.

### C. Conversion of prices to discount rates for Treasury bills of all maturities:

#### Formula:

$$d = \left[ \frac{100 - P}{100} \times \frac{360}{r} \right]$$

#### Where:

P = price per 100 (dollars).  
 d = discount rate.  
 r = number of days remaining to maturity.

#### Example:

For a 26-week bill issued December 30, 1982, due June 30, 1983, with a price of \$95.934567, solve for the discount rate (d).

#### Definitions:

P = 95.934567.  
 r = 182 (December 30, 1982, to June 30, 1983).

#### Resolution:

$$d = \left[ \frac{100 - P}{100} \times \frac{360}{r} \right]$$

$$d = \left[ \frac{100 - 95.934567}{100} \times \frac{360}{182} \right]$$

(2)  $d = [.04065433 \times 1.978021978]$ .

(3)  $d = .080415158$ .

(4)  $d = 8.042\%$ .

NOTE: Prior to April 18, 1983, we sold all bills in price-basis auctions, in which discount rates calculated from prices were rounded to three places, using normal rounding procedures. Since that time, we have sold bills only on a discount rate basis.

### D. Calculation of investment rate (coupon-equivalent yield) for Treasury bills:

1. For bills of not more than one half-year to maturity:

#### Formula:

$$i = \left[ \frac{100 - P}{P} \times \frac{y}{r} \right]$$

#### Where:

i = investment rate, in decimals.

## Pt. 356, App. C

P = price per 100 (dollars).

r = number of days remaining to maturity.

y = number of days in year following the issue date; normally 365 but, if the year following the issue date includes February 29, then y is 366.

Example:

For a cash management bill issued June 1, 1990, due June 21, 1990, with a price of \$99.559444 (computed from a discount rate of 7.930%), solve for the investment rate (i).

Definitions:

P = 99.559444.

r = 20 (June 1, 1990, to June 21, 1990).

y = 365.

Resolution:

$$i = \left[ \frac{100 - P}{P} \times \frac{y}{r} \right]$$

$$(1) i = \left[ \frac{100 - 99.559444}{99.559444} \times \frac{365}{20} \right]$$

$$(2) i = [.004425 \times 18.25].$$

$$(3) i = .080756.$$

$$(4) i = 8.076\%.$$

2. For bills of more than one half-year to maturity:

Formula:

$$P [1 + (r - y/2)(i/y)] (1 + i/2) = 100.$$

This formula must be solved by using the quadratic equation, which is:

$$ax^2 + bx + c = 0.$$

$$i = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$(1) i = \frac{-.997260274 + \sqrt{(.997260274)^2 - 4[(.248630137)(-.083834607)]}}{2(.248630137)}$$

$$(2) i = \frac{-.997260274 + \sqrt{.994528054 + .083375239}}{.497260274}$$

$$(3) i = (-.997260274 + 1.038221216) / .497260274.$$

$$(4) i = .040960942 / .497260274.$$

$$(5) i = .082373244 \text{ or}$$

$$(6) i = 8.237\%.$$

[69 FR 45202, July 28, 2004, as amended at 69 FR 52967, Aug. 30, 2004; 69 FR 53622, Sept. 2, 2004; 73 FR 14939, Mar. 20, 2008; 78 FR 46428 and 46430, July 31, 2013; 78 FR 50335, Aug. 19, 2013; 78 FR 52857, Aug. 27, 2013]

EDITORIAL NOTE: At 78 FR 59228-59230, Sept. 26, 2013, appendix B to part 356 was amended; however, portions of the amendment could not be incorporated due to inaccurate amendatory instructions.

## 31 CFR Ch. II (7-1-14 Edition)

Therefore, rewriting the bill formula in the quadratic equation form gives:

$$\left[ \frac{r}{2y} - .25 \right] i^2 + \left( \frac{r}{y} \right) i + \left( \frac{P - 100}{P} \right) = 0$$

and solving for “i” produces:

$$i = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

Where:

i = investment rate in decimals.

b = r/y.

a = (r/2y) - .25.

c = (P - 100)/P.

P = price per 100 (dollars).

r = number of days remaining to maturity.

y = number of days in year following the issue date; normally 365, but if the year following the issue date includes February 29, then y is 366.

Example:

For a 52-week bill issued June 7, 1990, due June 6, 1991, with a price of \$92.265000 (computed from a discount rate of 7.65%), solve for the investment rate (i).

Definitions:

r = 364 (June 7, 1990, to June 6, 1991).

y = 365.

P = 92.265000.

b = 364 / 365, or .997260274.

a = (364 / 730) - .25, or .248630137.

c = (92.265 - 100) / 92.265, or -.083834607.

Resolution:

## APPENDIX C TO PART 356—INVESTMENT CONSIDERATIONS

### I. INFLATION-PROTECTED SECURITIES

#### A. Principal and Interest Variability

An investment in securities with principal or interest determined by reference to an inflation index involves factors not associated with an investment in a non-indexed security. Such factors include the possibility that:

- The inflation index may be subject to significant changes,
- changes in the index may or may not correlate to changes in interest rates generally or with changes in other indices,